Liquidity Trap and Excessive Leverage*

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Abstract

We investigate the role of macroprudential policies in mitigating liquidity traps. When constrained households engage in deleveraging, the interest rate needs to fall to induce unconstrained households to pick up the decline in aggregate demand. If the fall in the interest rate is limited by the zero lower bound, aggregate demand is insufficient and the economy enters a liquidity trap. In this environment, households' *ex-ante* leverage and insurance decisions are associated with aggregate demand externalities. Welfare can be improved with macroprudential policies targeted towards reducing leverage. Interest rate policy is inferior to macroprudential policies in dealing with excessive leverage.

JEL Classification: E32, E4

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1 Introduction

Leverage has been proposed as a key contributing factor to the recent recession and the slow recovery in the US. Figure 1 illustrates the dramatic rise of leverage in the household sector before 2008 as well as the subsequent deleveraging episode. Using county-level data, Mian and Sufi (2012) have argued that household deleveraging is responsible for much of the job losses between 2007 and 2009. This view has recently been formalized in a number of theoretical models, e.g., Guerrieri and Lorenzoni (2011), Hall (2011), Eggertsson and Krugman (2012). These models have emphasized that deleveraging represents a reduction in aggregate demand. The interest rate needs to fall to induce unconstrained households to make up for the lost aggregate demand. However, the nominal interest rate cannot fall below zero given that hoarding cash provides an alternative to holding bonds—a phenomenon also known as the *liquidity trap*. When (expected) inflation is sticky, the lower bound on the nominal rate also prevents the real interest rate from declining, plunging the economy into a demand-driven recession. Figure 2 illustrates that the short term nominal and real interest rates in the US has indeed seemed constrained since December 2008.

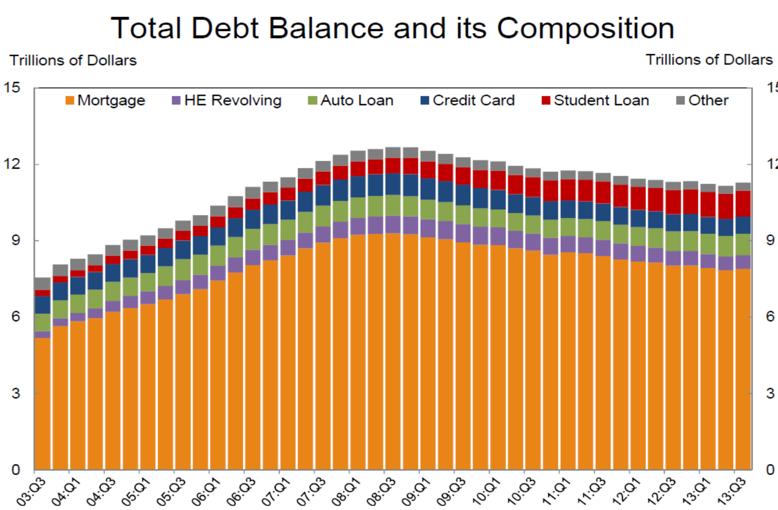

Figure 1: Evolution of household debt in the US over a window of +/− 5 years from its peak in 2008Q3. Source: Quarterly Report on Household Debt and Credit (August 2013), Federal Reserve Bank of New York.

An important question concerns the optimal policy to these types of episodes. The US Treasury and the Federal Reserve have responded to the recent recession by utilizing fiscal stimulus and unconventional monetary policies. These policies are (at least in part) supported by a growresponseing theoretical literature that emphasizes

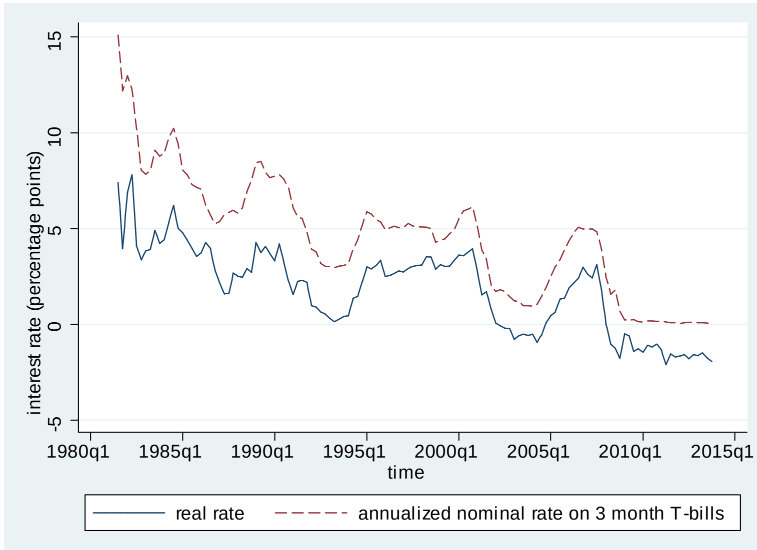

Figure 2: Nominal and real interest rates on 3 month US Treasury Bills between the third quarter of 1981 and the fourth quarter of 2013. The real interest rate is calculated as the annualized nominal rate minus the annualized current-quarter GDP inflation expectations obtained from the Philadelphia Fed's Survey of Professional Forecasters.

the benefits of stimulating aggregate demand during a liquidity trap. The theoretical contributions have understandably taken an ex-post perspective—characterizing the optimal policy once the economy is in the trap. Perhaps more surprisingly, both the practical and theoretical policy efforts have largely ignored the debt markets, even though the problems are thought to have originated in these markets.[1] In this paper, we analyze the scope for *ex-ante macroprudential policies* in debt markets—such as debt limits and insurance subsidies for deleveraging episodes.

To investigate optimal macroprudential policies, we present a tractable model, in which a tightening of borrowing constraints (e.g., due to a financial shock) leads to deleveraging and may trigger or contribute to a liquidity trap. The distinguishing feature of our model is that some households, which we call borrowers, endogenously accumulate leverage—even though households are aware that borrowing constraints will be tightened in the future. If borrowers have a sufficiently strong motive to borrow, e.g., due to impatience, then the economy features an anticipated deleveraging

[1] Several papers capture the liquidity trap in a representative household framework which leaves no room for debt market policies (see Eggertsson and Woodford (2003), Christiano et al. (2011), Werning (2012)). An exception is Eggertsson and Krugman (2011), which features debt but does not focus on debt market policies.

episode along with a liquidity trap.

Our main result is that it is desirable to slow down the accumulation of leverage in these episodes. In the run-up to deleveraging, borrowers who behave individually rationally undertake *excessive leverage* from a social point of view. Macroprudential policies that restrict leverage (coupled with appropriate ex-ante transfers) could make all households better off. This result obtains whenever deleveraging coincides with a liquidity trap—assuming that the liquidity trap cannot be fully alleviated by ex-post policies.

The mechanism behind the constrained inefficiency is an *aggregate demand externality* that applies in environments in which output is influenced by aggregate demand. When this happens, households' decisions that affect aggregate demand also affect aggregate output, and therefore other households' income. households do not take into account these general equilibrium effects, which may lead to inefficiencies. In our economy, the liquidity trap ensures that output is influenced by demand and that it is below its (first-best) efficient level. Moreover, greater ex-ante leverage leads to a greater ex-post reduction in aggregate demand and a deeper recession. This is because deleveraging transfers liquid wealth from borrowers to lenders, but borrowers who are constrained to delever have a much higher marginal propensity to consume (MPC) out of liquid financial wealth than lenders. Borrowers who choose their debt level (and lenders who finance them) do not take into account the negative demand externalities, leading to excessive leverage. In line with this intuition, we also show that the size of the optimal intervention, e.g., the optimal tax on borrowing, depends on the MPC differences between borrowers and lenders.

In practice, deleveraging episodes are often highly uncertain from an ex-ante point of view, as they are often driven by financial shocks such as a decline in collateral values. A natural question is whether households share the risk associated with deleveraging efficiently. Our second main result establishes that borrowers are also *underinsured* with respect to deleveraging episodes that coincide with a liquidity trap. Macroprudential policies that incentivize borrowers to take on more insurance can improve welfare. Intuitively, borrowers' insurance purchases transfer financial wealth during deleveraging from lenders (or insurance providers) to borrowers who have a higher MPC. This increases aggregate demand and mitigates the recession. Households do not take into account these aggregate demand externalities, which leads to too little insurance. The size of the optimal intervention, e.g., the optimal insurance subsidy, depends on borrowers' and lenders' MPC differences. An important financial shock in practice is a decline in house prices, which can tighten homeowners' borrowing constraints and trigger deleveraging. In this context, our results support policies that reduce homeowners' exposure to a decline in house prices, such as subsidies for home equity insurance.

While some financial shocks that induce deleveraging can be insured against (in principle), others might be much more difficult to describe and contract upon. We show that these types of environments with incomplete markets also feature excessive leverage in view of aggregate demand externalities. Welfare can be improved with "blanket" macroprudential policies that restrict non-contingent debt, as these provide protection against all deleveraging episodes, including those driven by uninsurable shocks. However, these policies also distort households' consumption in many future states without deleveraging, and thus, their optimal size depends on the ex-ante probability of deleveraging

We also investigate whether preventive monetary policies could be used to address aggregate demand externalities generated by leverage. A common argument is that a contractionary policy that raises the interest rate in the run-up to the recent subprime crisis could have been beneficial in curbing leverage. Perhaps surprisingly, our model reveals that raising the interest rate during the leverage accumulation phase can have the unintended consequence of *increasing* leverage. A higher interest rate reduces borrowers' incentives to borrow keeping all else equal—which appears to be the conventional wisdom informed by partial equilibrium reasoning. However, the higher interest rate also creates a temporary recession (or a slowdown in output growth) which increases borrowers' incentives to borrow so as to smooth consumption. In addition, the higher interest rate also transfers wealth from borrowers to lenders, which further increases borrowers' incentives to borrow. In our model, the general equilibrium effects can dominate under natural assumptions (e.g., when borrowers and lenders have the same intertemporal elasticity of substitution), and raising the interest rate can have the perverse effect of raising leverage. Our findings may explain why the interest rate hikes by the Fed starting in June 2004 were ineffective in reducing leverage at the time, as illustrated in Figures 1 and 2.

There are versions of our model in which the conventional wisdom holds, and raising the interest rate lowers leverage (as in Cúrdia and Woodford, 2009). But even in these cases, the interest rate policy is inferior to macroprudential policies in dealing with excessive leverage. Intuitively, constrained efficiency requires setting a wedge between borrowers' and lenders' *relative* interest rates, whereas the interest rate policy creates a different intertemporal wedge that affects all households' interest rates. As a by-product, the interest rate policy also generates an unnecessary slowdown in output growth—which is not a feature of constrained efficient allocations. That said, a different preventive monetary policy, namely raising the inflation target, is supported by our model as it would reduce the incidence of liquidity traps—and therefore, the relevance of aggregate demand externalities.

Our final analysis concerns endogenizing the debt limit faced by borrowers by assuming that debt is collateralized by financial assets, creating the potential for fire-sale effects. This introduces a new feedback loop into the economy, with two main implications. First, higher leverage lowers asset prices in the deleveraging phase, which in turn lowers borrowers' debt capacity and increases their distress. Hence, higher leverage generates fire-sale externalities that operate in the same direction as aggregate demand externalities. Second, an increase in borrowers' distress induces a more severe deleveraging episode and a deeper recession. Hence, fire-sale externalities exacerbate aggregate demand externalities. Conversely, lower aggregate output further lowers asset prices, exacerbating fire-sale externalities. These observations suggest that deleveraging episodes that involve asset fire-sales are particularly severe.

The remainder of this paper is structured as follows. The next subsection discusses the related literature. Section 2 introduces the key aspects of our environment. Section 3 characterizes an equilibrium that features an anticipated deleveraging episode that coincides with a liquidity trap. The heart of the paper is Section 4, which illustrates aggregate demand externalities, presents our main result about excessive leverage, and derives its policy implications. This section also relates the size of the optimal policy intervention to MPC differences between borrowers and lenders. Section 5 generalizes the model to incorporate uncertainty and presents our second main result about underinsurance. This section also generalizes the excessive leverage result to a setting with uncertain and uninsurable shocks. Section 6 discusses the role of preventive monetary policies in our environment. Section 7 presents the extension with endogenous debt limits and fire sale externalities, and Section 8 concludes. The appendix contains omitted proofs and derivations as well as some extensions of our baseline model.

1.1 Related literature

Our paper is related to a long economic literature studying the zero lower bound on nominal interest rates and liquidity traps, starting with Hicks (1937), and more recently emphasized by Krugman (1998) and Eggertsson and Woodford (2003, 2004). A growing recent literature has investigated the optimal fiscal and monetary policy response to liquidity traps (see e.g. Eggertsson, 2011; Christiano et al., 2011; Werning, 2012; Correia et al., 2013). Our contribution to this literature is that we take an ex-ante perspective, and focus on macroprudential policies in debt markets. We view this as an important exercise since the recent experience in a number of advanced economies suggests the set of policy instruments discussed in the cited literature was either restricted or insufficient to allow for a swift exit from the liquidity trap.

Guerrieri and Lorenzoni (2011) and Eggertsson and Krugman (2012) describe how financial market shocks that induce borrowers to delever lead to a decline in interest rates, which in turn can trigger a liquidity trap. Our framework is most closely related to Eggertsson and Krugman because we also model deleveraging between a set of impatient borrowers and patient lenders. They focus on the ex-post implications of deleveraging as well as the effects of monetary and fiscal policy during these episodes. Our contribution is to add an ex-ante stage and to investigate the role of macroprudential policies. Among other things, our paper calls for novel policy actions in debt markets that are significantly different from the more traditional policy responses to liquidity traps.

Our paper is part of a growing literature that investigates the role of macroprudential policies in mitigating financial crises. This literature has emphasized that agents can take excessive leverage or risks in view of moral hazard (e.g., Farhi and Tirole (2012), Gertler et al. (2012), Chari and Kehoe (2013)), neglected risks (e.g., Gennaioli at al. (2013)) or pecuniary externalities (e.g., Caballero and Krishnamurthy (2003), Lorenzoni (2008), Bianchi and Mendoza (2010), Jeanne and Korinek (2010ab) and Korinek (2011)). We show that aggregate demand externalities can also induce excessive leverage and risk taking, but through a very different channel. In contrast to pecuniary externalities, aggregate demand externalities apply not when prices are volatile, but in the opposite case when a certain price—namely the real interest rate—is fixed. We discuss the differences with pecuniary externalities further in Section 4, and illustrate the interaction of our mechanism with fire-sale externalities in Section 7.

The aggregate demand externality we focus on has first been discovered in the context of firms' price setting decisions, e.g., by Mankiw (1985), Akerlof and Yellen (1985) and Blanchard and Kiyotaki (1987). The broad idea is that, when output is not at its efficient level and influenced by aggregate demand, decentralized allocations that affect aggregate demand are socially inefficient. In Blanchard and Kiyotaki, output is not at the efficient level due to monopoly distortions, and firms' price setting affects aggregate demand due to complementarities in firms' demand. In our setting, output is below its efficient level due to the liquidity trap. We also focus on households' debt choices—as opposed to firms' price setting decisions—which affect aggregate demand due to differences in households' marginal propensities to consume.

A number of recent papers, e.g., Farhi and Werning (2012ab, 2015) and Schmitt-Grohe and Uribe (2011, 2012ab), also analyze aggregate demand externalities in contexts similar to ours. Schmitt-Grohe and Uribe analyze economies with fixed exchange rates that exhibit downward rigidity in nominal wages. They identify negative aggregate demand externalities associated with actions that increase wages during good times, because these actions lead to greater unemployment during bad times. In Farhi and Werning (2012ab), output responds to aggregate demand because prices are sticky and countries are in a currency union (and thus, under the same monetary policy). They emphasize the inefficiencies in cross-country insurance arrangements. In our model, output is demand-determined because of a liquidity trap, and we emphasize the inefficiencies in household leverage in a closed economy setting.

Farhi and Werning (2013) distill the broader lessons from this emerging literature on aggregate demand externalities in a general framework. They show that financial market allocations in economies with nominal rigidities that cannot be fully undone with monetary policy are generically inefficient. They also provide a number of general results for these environments, including optimal tax formulas to correct aggregate demand externalities. Our paper focuses on the inefficiencies in one specific setting, in a liquidity trap driven by deleveraging. We believe that this setting captures one of the most important occurrences of aggregate demand externalities in the US economy in recent decades. Farhi and Werning (2013) also analyze this setting as one out of several examples, which was developed independently and parallel to our work, but they do not provide an in-depth analysis. Our paper's unique analyses include the characterization of macroprudential policies with uncertainty (with and without incomplete markets) and the investigation of the effect of the interest rate policy on leverage.

Finally, our paper is also related to the recent New Keynesian literature that investigates the role of financial frictions and nominal rigidities in the Great Recession (see, for instance, Cúrdia and Woodford (2011), Gertler and Karadi (2011), Christiano, Eichenbaum and Trabandt (2014)). We share with this literature the view that financial frictions, combined with high leverage, can induce a demand-driven recession, especially if monetary policy is constrained by the zero lower bound. We differ in our emphasis on household leverage as opposed to financial institutions' (or firms') leverage. We also provide different and complementary remedies for the liquidity trap. While we emphasize macroprudential policies designed to correct externalities, this literature focuses on credit policies (e.g., lending or asset purchases by the central bank) that rely on the government's comparative advantage in financial intermediation (especially during a financial crisis). Both types of policies help to alleviate the liquidity trap, but they do so through different channels. Macroprudential policies prevent leverage from accumulating in the first place, whereas credit policies can be thought as containing the ex-post damage by slowing down deleveraging.

2 Environment and equilibrium

In this section, we introduce the key ingredients of the environment and define the equilibrium, which we characterize in subsequent sections.

Household debt and the anticipated financial shock The economy is set in infinite discrete time, with dates $t \in \{0, 1, ...\}$. There is a single consumption good, which is also the numeraire for real prices. There are two groups of households, borrowers and lenders, denoted by $h \in \{b, l\}$, with equal measure of each group normalized to 1. Households are symmetric except that borrowers have a weakly lower discount factor than lenders, $\beta^b \leq \beta^l < 1$, which will induce borrowers to take on debt in equilibrium. Let d_t^h denote the outstanding debt—or assets, if negative—of household h at date t. Households start with initial debt or asset levels denoted by d_0^h. At each date t, they face the one-period interest rate r_{t+1} and they choose their debt or asset levels for the next period, d_{t+1}^h.

Our first key ingredient is that, from date 1 onwards, households are subject to a borrowing constraint, that is, $d_{t+1}^h \leq \phi$ for each $t \geq 1$. Here, $\phi > 0$ denotes an exogenous debt limit as in Aiyagari (1994), or more recently, Eggertsson and Krugman (2012). The constraint can be thought of as capturing a financial shock in reduced from, e.g., a drop in collateral values or loan-to-value ratios, that would force households to reduce their leverage. In contrast, we assume that households can choose d_1^h at date 0 without any constraints. The role of these ingredients is to generate household leveraging at date 0 followed by deleveraging at date 1 along the lines of Figure 1. Moreover, to study the efficiency of households' ex-ante decisions, we assume that the deleveraging episode is anticipated at date 0. In our baseline model, we abstract away from uncertainty so that the episode is perfectly anticipated. We will introduce uncertainty in Section 5.1.

Households optimally choose their labor supply, in addition to making a dynamic consumption and saving decision. For the baseline model, we assume households' preferences over consumption and labor take the particular form $u\left(\tilde{c}_t^h - v\left(n_t^h\right)\right)$. These preferences provide tractability but are not necessary for our main results (see Appendix B.4). As noted in Greenwood, Hercowitz and Huffman (GHH, 1988), the specification implies that there is no income effect on the labor supply. Specifically, households' optimal labor supply solves the static optimization problem:

$$e_t \equiv \left(\max_{n_t^h} w_t n_t^h - v\left(n_t^h\right)\right) + \int_0^1 \Gamma_t(\nu)\, d\nu - T_t. \tag{1}$$

Here, e_t is the households' *net income*, that is, her total income net of labor costs. In addition to their labor income, households also symmetrically receive profits from firms that will be described below, $\int_0^1 \Gamma_t(\nu)\, d\nu$, and pay lump-sum taxes, T_t. Observe that (due to symmetry and the absence of income effects) households of each type will optimally supply the same amount of labor $n_t \equiv n_t^h$ and receive the same level of net income e_t.

Analogous to net income, we also define households' *net consumption* as their consumption net of labor costs, $c_t^h = \tilde{c}_t^h - v(n_t)$. Households' consumption and saving problem can then be written in terms of net variables as:

$$\begin{aligned} \max_{\left\{c_t^h, d_{t+1}^h\right\}_t} \quad & \sum_{t=0}^{\infty} \left(\beta^h\right)^t u\left(c_t^h\right) & (2) \\ \text{s.t. } c_t^h \quad &= e_t - d_t^h + \frac{d_{t+1}^h}{1+r_{t+1}} \text{ for all } t, \\ \text{and } d_{t+1}^h \quad &\leq \phi \text{ for each } t \geq 1. \end{aligned}$$

Problems (1) and (2) describe the optimal household behavior in our setting. We also make the standard assumptions about preferences: that is $u(\cdot)$ and $v(\cdot)$ are both strictly increasing, $u(\cdot)$ is strictly concave and $v(\cdot)$ is strictly convex, and they satisfy the conditions $\lim_{c\to 0} u'(c) = \infty$, $v'(0) = 0$ and $\lim_{n\to\infty} v'(n) = \infty$.

Liquidity trap and the bound on the nominal rate As we will see, household deleveraging will lower aggregate demand and put downward pressure on the interest rate. Our second key ingredient is a lower bound on the nominal interest rate. We assume there is cash (that is, paper money) in the economy that provides households with transaction services. To simplify the notation and the exposition, however, we consider the limit in which the transaction value of cash approaches zero (as described in Woodford (2003)). In the limit, the monetary authority still controls the short term nominal interest rate i_{t+1}. However, the presence of paper money (albeit a vanishingly small amount) sets a lower bound on the nominal interest rate,

$$i_{t+1} \geq 0 \text{ for each } t. \tag{3}$$

Intuitively, the nominal interest rate cannot fall significantly below zero, since households would otherwise hold cash instead of keeping their wealth in interest-bearing (or, more precisely, interest-charging) accounts.[2] A situation in which the nominal interest rate is at its lower bound is known as a liquidity trap. In a liquidity trap, cash and bonds become very close substitutes and households start demanding cash also for saving purposes. As this happens, increasing the money supply in the economy does not lower the nominal interest rate further since the additional money merely substitutes for bonds in households' portfolios.

Nominal rigidities and the bound on the real rate The bound on the nominal interest rate does not necessarily affect real allocations. Our third key ingredient is nominal rigidities, which turns the bound on the nominal rate into a bound on the real rate, with implications for real variables. We capture this ingredient by utilizing a standard New Keynesian model with an extreme form of price stickiness (see Remarks 1-3 below for a discussion and alternative specifications).

Specifically, suppose labor can be utilized to produce the consumption good via two types of firms. First, a competitive final good sector uses intermediate varieties $\nu \in [0, 1]$ to produce the consumption according to the Dixit-Stiglitz technology,

$$y_t = \left(\int_0^1 y_t(\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu \right)^{\varepsilon/(\varepsilon-1)} \text{ where } \varepsilon > 1, \tag{4}$$

where y_t denotes aggregate output per household. Second, a unit mass of monopolistic firms labeled by $\nu \in [0, 1]$ each produce $y_t(\nu)$ units of intermediate variety ν (per

[2] Recently, a number of central banks have cut interest rates to levels that are slightly below zero. The most prominent case was the Swiss National Bank (SNB) with a benchmark rate cut to –0.75% in January 2015. The cut was combined with regulations that exempted bank reserves held against small deposits from the negative rates since the SNB feared that small depositors would otherwise shift their holdings into currency. For larger depositors, the costs of holding large amounts of currency were viewed as sufficiently large to discourage significant shifts towards cash. However, policymakers expressed concerns that this may change once the negative rates would reach –1%. These events suggest that, even though the lower bound is not exactly zero in practice, it is arguably very close to zero.

household) by employing $n_t(\nu)$ units of labor according to the linear technology,

$$y_t(\nu) = n_t(\nu). \tag{5}$$

Let $P_t(\nu)$ denote the nominal price level for the monopolist for variety ν at time t. Given the Dixit-Stiglitz technology, the nominal price of the consumption good at time t is given by $P_t = \left(\int P_t(\nu)^{1-\varepsilon} d\nu\right)^{1/(1-\varepsilon)}$.

In our baseline model, we assume monopolists have preset nominal prices that are equal to each other and that never change, $P_t(\nu) = P$ for each t. This implies that the final good price is also constant, $P_t = P$ for each t. Combining this with (3) implies that the nominal and the real interest rates are the same. Consequently, the latter is also bounded form below:

$$i_{t+1} = r_{t+1} \geq 0 \text{ for each } t. \tag{6}$$

As Figure 2 illustrates, the real interest rate in the US in recent years indeed seems bounded from below. We normalize inflation to zero so that the lower bound on the real rate is also zero. Appendix B.1 shows that our results are qualitatively robust to allowing for a higher yet sticky inflation rate.

Demand determined output and constrained monetary policy Our fourth and final ingredient is that, when the interest rate is at its lower bound, the economy experiences a demand-driven recession. To introduce this ingredient, we first describe the efficient allocations in this environment. Given the linear technology in (4) and (5), and the household preferences in (1), the efficient level of net income and labor supply are respectively given by:

$$e^* \equiv \max_{n_t} n_t - v(n_t) \text{ and } n^* \equiv \arg\max_{n_t} n_t - v(n_t). \tag{7}$$

We next describe a frictionless benchmark without price rigidities, which also generates the efficient allocations. Suppose each monopolist resets its price every period. The monopolist faces isoelastic demand for its goods, $y_t p_t(\nu)^{-\varepsilon}$, where $p_t(\nu) = P_t(\nu)/P_t$ denotes its relative price. Thus, her problem can be written (in terms of per household variables) as:

$$\Gamma_t(\nu) = \max_{p_t(\nu), y_t(\nu), n_t(\nu)} p_t(\nu) y_t(\nu) - w_t [1-\tau(n_t)] n_t(\nu) \text{ s.t. } y_t(\nu) = n_t(\nu) \leq y_t p_t(\nu)^{-\varepsilon}. \tag{8}$$

Here, $\tau(n_t)$ captures linear subsidies to employment of each monopolist ν, which are financed by lump-sum taxes, that is, $T_t = \tau(n_t) w_t \int_0^1 n_t(\nu) d\nu$. We assume these subsidies in order to correct the distortions that arise from monopolistic competition and focus our welfare analysis solely on aggregate demand externalities. Specifically, we set $\tau(n_t) = 1/\varepsilon$ if the aggregate employment is below the efficient level, $n_t \leq n^*$, and $\tau(n_t) = 0$ otherwise. The optimality conditions for problems (8) and (1) then imply $e_t = e^*$ for each t. Thus, the subsidies provide us with an efficient benchmark for welfare comparisons, although they are not necessary for any of our results (see Appendix B.3 for the case with $\tau = T_t = 0$, as well as an explanation for why we take away the subsidies when $n_t > n^*$).

Set against this frictionless benchmark, monopolistic firms in our setting have the preset nominal price $P_t(\nu) = P$. Their optimization problem can then be written as

$$\Gamma_t(\nu) = \max_{y_t(\nu), n_t(\nu)} p_t(\nu) y_t(\nu) - w_t (1 - \tau(n_t)) n_t(\nu) \text{ s.t. } y_t(\nu) = n_t(\nu) \leq y_t p_t(\nu)^{-\varepsilon}, \tag{9}$$

where $p_t(\nu) = P_t(\nu)/P$ denotes the monopolist's fixed relative price, which is equal to 1 by symmetry. That is, the monopolist chooses how much to produce subject to the constraint that its output cannot exceed the demand for its goods. In the equilibria we analyze, the monopolist always meets the demand for its goods, $y_t(\nu) = n_t(\nu) = y_t$, since its marginal cost is strictly below its price. By symmetry, this induces an equilibrium level of employment $n_t = y_t$ and net income $e_t = y_t - v(y_t)$.

It follows that the outcomes in this model are ultimately determined by the aggregate demand (per household) for the final consumption good, $y_t = \frac{\tilde{c}_t^b + \tilde{c}_t^l}{2}$. This in turn depends on monetary policy, which controls the nominal and the real interest rate. Since the price level is fixed, we assume that the monetary policy focuses on myopic output stabilization (analogous to a Taylor rule) subject to the constraint in (6). In our setting, this amounts to replicating the frictionless benchmark, by setting:

$$i_{t+1} = r_{t+1} = \max\left(0, r_{t+1}^*\right) \text{ for each } t. \tag{10}$$

Here, r_{t+1}^* is recursively defined as the frictionless interest rate at time t that obtains when households' net income is $e_t = e^*$ and the monetary policy follows the rule in (10) at all future dates $\tilde{t} \geq t+1$. This policy is also constrained efficient in our environment, as long as the monetary authority does not have commitment power (see Appendix A.1).[3]

[3] A monetary authority with commitment power might find it desirable to deviate from (10) by

Definition 1 (Equilibrium). *The equilibrium is a path of real allocations,* $\left\{\left[n_t^h, c_t^h, d_{t+1}^h\right]_h, e_t, y_t, \left[y_t\left(\nu\right), n_t\left(\nu\right)\right]_\nu\right\}_t$*, and wages, interest rates, profits, and taxes* $\left\{w_t, r_{t+1}, \left[\Gamma_t\left(\nu\right)\right]_\nu, T_t\right\}_t$*, such that the households' allocations solve problems* (1) *and* (2)*, a competitive final good sector produces according to* (4)*, the intermediate good monopolists solve* (9) *for given fixed goods prices, the interest rates are set according to* (10)*, and all markets clear.*

Remark 1 (Interpretation of Price Stickiness). We interpret our extreme price stickiness assumption as capturing in reduced form an environment in which *the aggregate price level* is sticky in the *upward direction* throughout the deleveraging episode. This ensures that the economy cannot have much inflation in the short run, which converts the bound on the nominal interest rate into a bound on the real rate as in (6). Our model is consistent with (at least) two forces that might contribute to upward price stickiness in practice: (i) price stickiness at the micro level and (ii) constraints on monetary policy against creating inflation. These forces, which are not mutually exclusive, can be isolated by considering the following two scenarios.

First, the prices at the micro level can be effectively very sticky (for reasons emphasized in the New Keynesian literature), as in a literal interpretation of our baseline model. In this case, the aggregate price level will also be very sticky in the short run, even if the monetary policy can flexibly react to the liquidity trap.

Second, prices may be somewhat flexible, but the monetary authority may be constrained to follow an inflation targeting policy with a predetermined target. Appendix B.1 analyzes this case and shows that the equilibrium features *the same real allocations* as in the baseline model (up to a log-linear approximation) if the inflation target is normalized to zero. Intuitively, even though there is some price flexibility at the micro level, the aggregate price level remains sticky in the upward direction due to the inflation targeting policy. In practice, many central banks follow policies along these lines, in view of their legal mandates to pursue price stability. Moreover, deviating from these policies so as to create inflation would be dynamically inconsistent. If inflation is costly, then the central bank would optimally revert to an inflation targeting policy once the economy exits the liquidity trap [see Werning (2012) for a formal analysis].

Remark 2 (Disinflation). Appendix B.1 also shows that once we introduce limited price flexibility, inflation falls below its target level during the liquidity trap (between

setting the interest rate below the frictionless benchmark after the economy exits a liquidity trap (see Werning, 2012). We abstract away from these "forward guidance" policies which are not our focus.

dates 0 and 1) in view of the negative output gap. This disinflation could further exacerbate the recession by tightening the bound on the real rate in (6).[4] It is perhaps fortunate that the US economy avoided a severe disinflation during the recent macroeconomic slump. A number of papers have argued that the "missing disinflation" represents a puzzle for the standard New Keynesian model and its Phillips Curve (e.g., Ball and Mazumder, 2011; Hall, 2013; Coibion and Gorodnichenko, 2015). More recent work, however, has found that the missing disinflation can be reconciled with the New Keynesian model (e.g., Del Negro et al. (2015)), especially after accounting for temporary factors such as the recent productivity slowdown or the financial constraints on firms during the crisis (e.g., Christiano et al. (2014), and Gilchrist et al. (2015)).

Remark 3 (Alternative Formulations for the Supply Side). We adopt a New Keynesian model with price rigidities in the goods market for expositional simplicity. However, our results are robust to several alternative specifications for the supply side. Appendix B.2 illustrates this point by analyzing a version of our model in which the nominal wages are rigid in the downward direction, as in Eggertsson and Mehrotra (2014) or Schmitt-Grohe and Uribe (2012c), whereas nominal prices are flexible.[5] In this formulation, the demand shortage due to the constraint in (B.4) is absorbed by rationing in the labor market—as opposed to rationing (or higher markups) in the goods market which then lowers employment.[6] Appendix B.2 shows that this formulation also yields *the same real allocations* as our baseline model, as long as we continue to assume an inflation targeting monetary policy.

3 An anticipated deleveraging episode

This section characterizes the decentralized equilibrium and describes an anticipated deleveraging episode that triggers a liquidity trap. The next section analyzes the

[4] This does not happen in our model due to the special feature that deleveraging takes place in a single period (between dates 1 and 2). If we were to split this episode into multiple sub-periods, then disinflation would induce a tighter bound on the real rate and a more severe recession during the earlier sub-periods (similar to Werning, 2012).

[5] Our NBER working paper illustrates yet another alternative formulation for the supply side based on an older rationing equilibrium concept, which has also been adopted in some recent work, e.g., Hall (2011), Kocherlakota (2012), and Caballero and Farhi (2013).

[6] In general, having rationing in the labor market (as opposed to the goods market) could exacerbate the recession, as it would make it more difficult for borrowers to pay back their debt by raising their labor supply. This difference does not show up in our setting because of the GHH preferences that shut down labor supply responses (see Eq. (1)).

efficiency properties of this equilibrium. We start with the following lemma, which describes the possibilities for equilibrium within a period.

Lemma 1. *(i) If $r_{t+1} > 0$, then $e_t = e^*$, (ii) If $r_{t+1} = 0$, then $e_t = \frac{c_t^b + c_t^l}{2} \leq e^*$.*

The first part captures the scenario in which the monetary policy in (10) replicates the frictionless outcome. The second part captures a liquidity trap scenario in which the frictionless outcome would call for a negative interest rate. In this case, the interest rate is constrained $r_{t+1} = 0$, and the economy experiences a demand-driven recession. Net income is below its frictionless level e^*, and is determined by net aggregate demand, $\frac{c_t^b + c_t^l}{2}$.

We next combine Lemma 1 with the households' consumption and savings problem (2) to characterize the full equilibrium. Note that the market clearing for debt implies $d_t^l = -d_t^b$. Therefore, we drop superscripts and let $d_t \equiv d_t^b$ denote the aggregate debt level in the economy. We will focus on cases in which borrowers' constraint binds at all dates, that is, $d_{t+1} = \phi$ for each $t \geq 1$. Throughout, we also make the following parametric assumptions.

Assumption (1). *(i)* $\frac{u'(2e^*)}{u'\left(e^* + \phi\left(1 - \beta^l\right)\right)} < \beta^l$, (ii) $d_0 < \tilde{d}_0$ (see Appendix A.1 for $\tilde{d}_0$).

The first part allows for the interest rate constraint (6) to bind at date 1, while the second part ensures that it doesn't bind at date 0, simplifying the exposition.

Steady state We characterize the equilibrium backwards. First consider dates $t \geq 2$, at which the outstanding debt level is already lowered to ϕ. At these dates, the economy is in a steady-state. Since borrowers are constrained, the real interest rate is determined by lenders' discount rate, $r_{t+1} = 1/\beta^l - 1 > 0$. Since the interest rate is positive, the economy features the frictionless outcomes [cf. Lemma 1]. In particular, households' consumption is given by:

$$c_t^b = e^* - \phi\left(1 - \beta^l\right) \quad \text{and } c_t^l = e^* + \phi\left(1 - \beta^l\right) \text{ for } t \geq 2. \tag{11}$$

Deleveraging Next consider date $t = 1$. Borrowers' consumption is given by $c_1^b = e_1 - \left(d_1 - \frac{\phi}{1+r_2}\right)$. Note that the larger the outstanding debt level d_1 is relative to the debt limit, the more borrowers are forced to reduce their net consumption. The resulting slack in aggregate demand needs to be absorbed by an increase in lenders' net consumption:

$$c_1^l = e^* + \left(d_1 - \frac{\phi}{1 + r_2}\right).$$

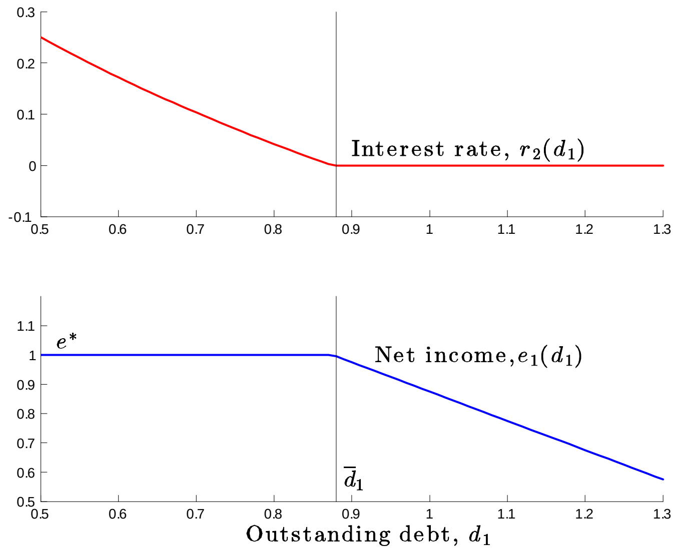

Figure 3: Interest rate and net income at date 1 as a function of outstanding debt d_1.

Since lenders are unconstrained, their Euler equation holds $\frac{u'\left(c_1^l\right)}{\beta^l u'\left(c_2^l\right)} = 1 + r_2$, where $c_2^l = e^* + \phi\left(1 - \beta^l\right)$. Hence, the increase in lenders' consumption at date 1 is mediated through a decrease in the real interest rate, r_2. The key observation is that the lower bound on the real interest rate effectively sets an upper bound on lenders' (or unconstrained agents') consumption in equilibrium, $c_1^l \leq \bar{c}_1^l$, given by the solution to

$$u'\left(\bar{c}_1^l\right) = \beta^l u'\left(e^* + \phi\left(1 - \beta^l\right)\right). \tag{12}$$

The equilibrium at date 1 then depends on the relative size of two terms:

$$d_1 - \phi \lesseqgtr \bar{c}_1^l - e^*.$$

The left hand side is the amount of deleveraging borrowers are forced into given that the borrowing limit falls to ϕ (and the real rate is at its lower bound). The right hand side is the maximum amount of demand the unconstrained agents can absorb when the real rate is at its lower bound. If the left side is smaller than the right side, then the equilibrium features $r_2 > 0$ and $e_1 = e^*$. In this case, the effects of deleveraging on aggregate demand is offset by a reduction in the real interest rate. The left side of Figure 3 (the range with $d_1 \leq \bar{d}_1$) illustrates this outcome.

Otherwise, equivalently if the debt level is strictly above a threshold,

$$d_1 > \bar{d}_1 = \phi + \bar{c}_1^l - e^*, \tag{13}$$

then the economy is in a liquidity trap. The real interest rate is at its lower bound, $r_2 = 0$ and the economy experiences a recession driven by low demand. Borrowers' and lenders' net consumption demand are respectively given by $c_1^b = e_1 - d_1 + \phi$ and $c_1^l = \bar{c}_1^l$. By Lemma 1, this implies,

$$e_1 = \frac{c_1^b + c_1^l}{2} = \frac{e_1 - (d_1 - \phi) + \bar{c}_1^l}{2}. \tag{14}$$

After rearranging this expression, the equilibrium level of net income is given by,

$$e_1 = \bar{c}_1^l + \phi - d_1 < e^*. \tag{15}$$

The right side of Figure 3 (the range with $d_1 \geq \bar{d}_1$) illustrates this outcome.

Eq. (14) illustrates that is a Keynesian cross and a Keynesian multiplier in our setting. Net income is equal to net aggregate demand as in a typical Keynesian cross. Each additional unit of debt reduces borrowers' net demand by half a unit because the share of borrowers in the population is 1/2 and their marginal propensity to consume (MPC) out of liquid wealth is 1 since they are constrained. This triggers a Keynesian multiplier: the decline in net demand reduces borrowers' net income by 1/2 unit, which in turn reduces the net demand further by 1/4 units, and so on. Eq. (15) puts these effects together and shows that an increase in outstanding debt leads to a deeper recession.

Intuitively, an increase in debt reduces demand and output by transferring wealth from borrowers that have a very high MPC out of liquid wealth to lenders that have a low MPC. The feature that borrowers' MPC is equal to 1 enables us to illustrate our inefficiency results sharply, but it is not necessary. Section 4.3 shows that net income is declining in outstanding debt, $\frac{\mathbf{d}e_1}{\mathbf{d}d_1} < 0$, as long as borrowers' MPC is greater than lenders' MPC. As we will see, this feature is all we need for aggregate demand externalities to be operational and to generate inefficiencies.

Date 0 Allocations We next turn to households' financial decisions at date 0. We conjecture an equilibrium in which the net income is at its efficient level, $e_0 = e^*$.

Since households are unconstrained at date 0, their Euler equations hold,

$$\frac{1}{1+r_1} = \frac{\beta^l u'\left(c_1^l\right)}{u'\left(c_0^l\right)} = \frac{\beta^b u'\left(c_1^b\right)}{u'\left(c_0^b\right)}. \tag{16}$$

The equilibrium debt level, d_1, and the interest rate, r_1, are determined by these equations. We next identify two conditions under which households choose a sufficiently high debt level that triggers a recession at date 1, $d_1 > \bar{d}_1$. The relevant thresholds, $\bar{\beta}^b(d_0)$ and $\bar{d}_0\left(\beta^b\right)$, are characterized in Appendix A.1.

Proposition 1. *There is an equilibrium with a deleveraging-induced recession at date* 1 *if the borrower is sufficiently impatient or sufficiently indebted at date* 0*. Specifically, for any debt level* d_0 *there is a threshold level of impatience* $\bar{\beta}^b(d_0)$ *such that the economy experiences a recession at date* 1 *if* $\beta^b < \bar{\beta}^b(d_0)$*. Conversely, for any level of impatience* β^b *there is a threshold debt level* $\bar{d}_0\left(\beta^b\right)$ *such that the economy experiences a recession at date* 1 *if* $d_0 > \bar{d}_0\left(\beta^b\right)$*.*

Proposition 1 describes two scenarios that might induce borrowers to carry a high level of debt into date 1, even though they anticipate the deleveraging episode as well as the associated liquidity trap. First, borrowers might have a sufficiently strong motive to borrow at date 0 (due to various spending opportunities) as captured by a low discount factor in our setting. Second, borrowers might also have accumulated a large amount of debt in the past, perhaps at a time at which they did not anticipate the deleveraging episode. We view both scenarios as relevant for macroprudential policy analysis in practice. The first scenario is useful to investigate whether the economy accumulates leverage optimally, and the second scenario is useful to analyze whether the economy can efficiently manage a "smooth landing" to low leverage.[7]

[7] While we emphasize deleveraging as the main cause of the liquidity trap, our welfare analysis is consistent with other forces that might also lower demand at date 1 such as the financial crisis (see, for instance, Gertler and Karadi, 2014) or investment overhang (see Rognlie, Shleifer, Simsek, 2014). In fact, these forces would be complementary to deleveraging in the sense that they would modify the thresholds in Proposition 1 so as to make a liquidity trap more likely. The important point for our welfare analysis is that deleveraging coincides with a liquidity trap. Recent work, e.g., Summers (2013) and Eggertsson and Mehrotra (2014), has also emphasized long-run forces that could have permanently reduced the aggregate demand as well as the safe interest rates. These forces are also complementary to our analysis, and they also suggest that deleveraging episodes and liquidity traps might continue to be a serious concern for the world economy in upcoming years.

4 Excessive leverage

We next analyze the efficiency properties of the equilibrium characterized in Proposition 1 and present our main result. We first illustrate the aggregate demand externalities in our setting, and contrast them with pecuniary externalities. We then illustrate that the competitive equilibrium is constrained inefficient and that it can be Pareto improved with simple macroprudential policies. The last part quantifies the size of the inefficiency, as well as the optimal intervention, in terms of households' MPC differences.

4.1 Aggregate demand externalities

We consider a constrained planner at date 0 that can affect the amount the aggregate debt level d_1 at date 1 (symmetrically held by borrowers) through policies we will describe but cannot interfere thereafter. We focus on constrained efficient allocations with $d_1 \geq \phi$, so that conditional on d_1, the economy behaves as we analyzed in the previous section for date 1 onwards.

Let $V^h\left(d_1^h; d_1\right)$ denote the utility of household h conditional on entering date 1 with an individual level of debt d_1^h, and an aggregate level of debt d_1. The aggregate debt enters household utility separately because it determines the interest rate or net income at date 1. More specifically, we have:

$$\begin{aligned} V^b\left(d_1^b, d_1\right) &= u\left(e_1\left(d_1\right) - d_1^b + \frac{\phi}{1 + r_2\left(d_1\right)}\right) + \sum_{t=2}^{\infty}\left(\beta^b\right)^t u\left(c_t^b\right) \\ V^l\left(d_1^l, d_1\right) &= u\left(e_1\left(d_1\right) - d_1^l - \frac{\phi}{1 + r_2\left(d_1\right)}\right) + \sum_{t=2}^{\infty}\left(\beta^l\right)^t u\left(c_t^l\right) \end{aligned} \tag{17}$$

where $r_2\left(d_1\right)$ and $e_1\left(d_1\right)$ are characterized in the previous section and the continuation utilities from date 2 onwards do not depend on d_1^h or d_1 [cf. Eq. (11)].

In equilibrium, we have $d_1^b = d_1 = -d_1^l$ in view of symmetry and market clearing. But taking d_1 explicitly into account is useful to illustrate the externalities. Specifically, raising the equilibrium debt level by one unit induces an uninternalized welfare effect $\frac{\mathbf{d}V^h}{\mathbf{d}d_1}$ on household h, which we characterize next.

Lemma 2. *(i) If* $d_1 \in [\phi, \bar{d}_1)$, *then* $\frac{\mathbf{d}V^h}{\mathbf{d}d_1} = \begin{cases} -\eta u'\left(c_1^h\right) < 0, \ \textit{if } h = l \\ \eta u'\left(c_1^h\right) > 0, \ \textit{if } h = b \end{cases}$, *where* $\eta \in (0, 1)$.

(ii) If $d_1 > \bar{d}_1$, then

$$\frac{\mathbf{d}V^h}{\mathbf{d}d_1} = \frac{\mathbf{d}e_1}{\mathbf{d}d_1} u'\left(c_1^h\right) = -u'\left(c_1^h\right) < 0, \text{ for each } h \in \{b, l\}. \tag{18}$$

The first part illustrates the usual pecuniary externalities on the interest rate, which apply when the debt level is relatively low. In this case, a higher debt level translates into a lower interest rate r_1—so as to counter the decline in demand—but it does not affect the net income, $e_1(d_1) = e_1^*$ (see Figure 3). The reduction in the interest rate generates a redistribution from lenders to borrowers captured by η (characterized in Eq. (A.4) in the appendix). Consequently, deleveraging imposes positive pecuniary externalities on borrowers but negative pecuniary externalities on lenders. In fact, since markets between date 0 and 1 are complete, these two effects "net out" from an ex-ante point of view: that is, the date 0 equilibrium is constrained Pareto efficient in this region (see Proposition 2).

The second part of the lemma illustrates the novel force in our model, *aggregate demand externalities*. In this case, the debt level is sufficiently large so that the economy is in a liquidity trap, which has two implications. First, the interest rate is fixed, $r_2(d_1) = 0$, so that the pecuniary externalities do not apply. Second, net income is decreasing in debt, $\frac{\mathbf{d}e_1}{\mathbf{d}d_1} < 0$, through a reduction in aggregate demand (see Figure 3). Consequently, an increase in aggregate debt reduces households' welfare, which we refer to an aggregate demand externality.

Lemma 2 also illustrates that, unlike pecuniary externalities, aggregate demand externalities hurt *all households*, because they operate by lowering incomes. This feature suggests that aggregate demand externalities can be considerably more potent than pecuniary externalities. They also lead to constrained inefficiencies in our setting, which we verify next.

4.2 Excessive leverage

We next show that the competitive equilibrium allocation can be Pareto improved by reducing leverage. One way of doing this is ex-post, by writing down borrowers' debt. To see this, suppose the planner reduces borrowers' outstanding debt to lenders from d_1 to the threshold, $\bar{d}_1$, given by Eq. (13). By our earlier analysis, the recession is avoided, and net income increases to its efficient level, e^*. Borrowers' net consumption and welfare naturally increase after this intervention. Less obviously, lenders' net consumption remains the same at the upper bound, $\bar{c}_1^l$. The debt write-down has a direct negative effect on lenders' welfare by reducing their assets, as captured by $\frac{-\mathbf{d}V^l}{\mathbf{d}d_1^l} = u'\left(c_1^l\right) > 0$. However, the debt write-down also has an indirect positive effect on lenders' welfare through aggregate demand externalities. Lemma 2 shows that the externalities are sufficiently strong to fully counter the direct effect, $\frac{-\mathbf{d}V^l}{\mathbf{d}d_1} = u'\left(c_1^l\right) > 0$, leading to an ex-post Pareto improvement.

From the lens of our model, debt write-downs are always associated with aggregate demand externalities. However, these externalities are not always sufficiently strong to lead to a Pareto improvement.[8] Furthermore, ex-post debt write-downs are difficult to implement in practice for a variety of reasons, e.g., legal restrictions, concerns with moral hazard, or concerns with the financial health of intermediaries (assuming that some lenders are intermediaries). Therefore we do not analyze our results on ex-post inefficiency further.

An alternative, and arguably more practical, way to reduce leverage is to prevent it from accumulating in the first place. This creates a very general scope for Pareto improvements. To investigate ex-ante optimality, suppose the planner can choose households' allocations at date 0, in addition to controlling the equilibrium debt level carried into date 1 (through the policies we will describe). We say that an allocation $\left(\left(c_0^h, n_0^h\right)_h, d_1\right)$ is *constrained efficient* if it is optimal according to this planner, that is, if it solves

$$\max_{\left(\left(c_0^h, n_0^h\right)_h, d_1\right)} \sum_h \gamma^h \left[u\left(c_0^h\right) + \beta^h V^h\left(d_1^h, d_1\right)\right] \tag{19}$$

$$\text{such that} \qquad d_1 = d_1^b = -d_1^l \text{ and } \sum_h c_0^h = \sum_h \left[n_0^h - v(n_0^h)\right].$$

Here, $\gamma^h > 0$ captures the relative welfare weight assigned to group h households. We next characterize the constrained efficient allocations over the relevant range.[9]

Proposition 2 (Optimal Leverage)**.** *An allocation $\left(\left(c_0^h, n_0^h\right)_h, d_1\right)$, with $d_1 \geq \phi$ and $u'\left(c_0^l\right) \geq \beta^l u'\left(c_1^l\right)$, is constrained efficient if and only if net income at date 0 is at its*

[8] For instance, with separable preferences, $u(c) - v(l)$, analyzed in Appendix B.4, debt writedowns do not generate ex-post Pareto improvement. This is also the case for the extension analyzed in Section 4.3 with flexible MPC differences.

[9] We restrict attention to solutions that satisfy $d_1 \geq \phi$ and $u'\left(c_0^l\right) > \beta^l u'\left(c_1^l\right)$, which is the relevant range of comparison with the competitive equilibrium characterized in Section 3. The former condition ensures that the exogenous debt limit also binds for the planner's allocation. The latter condition ensures that the planner's allocation can be implemented without hitting the zero lower bound at date 0, i.e., in the period before the deleveraging. Note that the competitive equilibrium features $r_1 > 0$ and satisfies this condition in view of Assumption (1).

frictionless level, i.e., $e_0 = e^*$*; and the consumption and debt allocations satisfy one of the following:*

(i) $d_1 \leq \bar{d}_1$ *and the Euler equations (16) hold.*

(ii) $d_1 = \bar{d}_1$ *and the following inequality holds:*

$$\frac{\beta^l u'\left(c_1^l\right)}{u'\left(c_0^l\right)} > \frac{\beta^b u'\left(c_1^b\right)}{u'\left(c_0^b\right)}. \tag{20}$$

The first part illustrates that competitive equilibrium allocations in which $d_1 \leq \bar{d}_1$ are constrained efficient. This part verifies that pecuniary externalities alone do not generate inefficiencies in our setting. The second part, which is our main result, shows that the planner never chooses a debt level above $\bar{d}_1$ that triggers a recession at date 1. Instead, the planner distorts decentralized households' Euler equations according to (20). At these allocations, borrowers would like to increase borrowing—so as to increase their consumption at date 0 and reduce their consumption at date 1—but they are prevented from doing so by the planner. In particular, a competitive equilibrium that features $d_1 > \bar{d}_1$, as well as the Euler equations (16), is constrained inefficient, as we formalize next.

Corollary 1 (Excessive Leverage)**.** *The competitive equilibrium allocation* $\left((c_0^{h,eq}, n_0^{h,eq})_h, d_1^{eq}\right)$ *in Proposition 1 is constrained inefficient and is Pareto dominated by the constrained efficient allocation* $\left(c_0^h, n_0^h\right)_h = (c_0^{h,eq}, n_0^{h,eq})_h$ *and* $d_1 = \bar{d}_1$*.*

To understand the intuition for the inefficiency, observe that lowering debt when the economy is in a liquidity trap generates first-order welfare benefits because of aggregate demand externalities, as illustrated in Lemma 2. By contrast, distorting agents' consumption away from their privately optimal levels generates locally second order losses. Thus, starting from an unconstrained equilibrium, it is always socially desirable to lower leverage. As this intuition suggests, the ex-ante inefficiency from excessive leverage applies quite generally. For instance, Appendix B.4 establishes an analogous result for the case with separable preferences, $u(c) - v(n)$, and Section 4.3 generalizes the result to the case in which borrowers have lower MPCs.[10]

We next show that the constrained efficient allocations in Proposition 2 and Corollary 1 can be implemented using simple macroprudential policies. We spell out two

[10] In our baseline setting, the externalities are so strong that the planner fully avoids a recession. This feature is less general. In Appendix B.4 and Section 4.3, the planner typically alleviates but does not fully eliminate the recession.

alternative implementations using quantity and price interventions in households' financial decisions. We allow the planner to use lump-sum transfers at date 0, which enables her to trace the constrained Pareto frontier characterized in Proposition 2.

Corollary 2 (Implementing the Optimal Leverage)**.** *The constrained efficient allocations characterized in Proposition 2 can be implemented alternatively with:*

(i) the debt limit $d_1^h \leq \bar{d}_1$ applied to all households, or

(ii) a tax $\tau_0^b \geq 0$ applied on any positive debt issuance $d_1^h > 0$ (that is rebated lump-sum to households), which satisfies,[11]

$$\frac{\beta^l u'\left(c_1^l\right)}{u'\left(c_0^l\right)} = \frac{\beta^b u'\left(c_1^b\right)}{u'\left(c_0^b\right)} \frac{1}{1-\tau_0^b}, \tag{21}$$

combined in each case with an appropriate lump-sum transfer $T_0^b \gtrless 0$ between borrowers and lenders.

The debt limit policy directly restricts the equilibrium debt level. The tax policy brings about the same outcome by raising borrowers' net-of-tax interest rate, $\frac{1+r_1}{1-\tau_0^b}$, relative to the lenders' rate, $1+r_1$. Note also that both of these policies are anonymous in the sense that they apply to all households. For lenders, the limit defined in (i) does not bind, and the tax rate in (ii) does not apply, because their debt issuance is negative. However, this feature of the model does not generalize to richer settings. In general, the optimal policy requires targeted interventions for different groups (see, for instance, Sections 4.3 and 5.1).

The corollary describes restrictions on borrowing, but observe that the same allocations can be implemented by policy measures on saving. A binding quantity limit on wealth accumulation $d_1^h < -\bar{d}_1$ would ensure that lenders will not carry excessive wealth into the deleveraging period. Similarly, a tax on wealth accumulation could achieve the same objective.[12]

More broadly, our analysis supports policies that are targeted towards lowering household leverage (as well as corporate and bank leverage, as we discuss in Section

[11] Specifically, a household who issues $d_1^h > 0$ units of debt at interest rate r_1 at date 0 receives only $\frac{1-\tau_0^b}{1+r_1} d_1^h$ units, whereas its lender needs to provide $\frac{1}{1+r_1} d_1^h$ units. The difference, $\frac{\tau_0^b}{1+r_1} d_1^h$, is government revenue, which is rebated lump-sum and equally to all households.

[12] One interesting further question is whether the planner's optimal intervention would change if the deleveraging is anticipated several periods in advance. We find that the optimal interventions are unchanged in this case. For instance, the planner could announce the debt limit for date 1 ($d_1^h \leq \bar{d}_1$) ahead of time, and let private agents decide how to optimally smooth consumption in earlier periods.

8). This is in contrast with some tax policies in the US, e.g., mortgage interest tax deduction, that incentivize households to take on debt. Our analysis provides another rationale for revisiting these policies, especially in environments (with already low interest rates) in which deleveraging can induce or exacerbate a liquidity trap.

Our findings also point out that macroeconomic stabilization and financial stabilization are two sides of the same coin in the described setup. Since the recession in our model is driven by deleveraging, macroprudential policies increase both macroeconomic stability (by mitigating recessions) and financial stability (by reducing the size of deleveraging). We employ the label "macro-prudential" for this policy since it constitutes a financial market intervention that delivers the macroeconomic benefit of avoiding output costs, in line with the ultimate objective of macroprudential policy described by Borio (2003).[13]

4.3 Quantifying the inefficiency with MPC differences

Let MPC_1^h denote the increase in household h's consumption at date 1 in response to a transfer of one unit of liquid wealth at date 1, keeping her wage and interest rates at all dates constant. Our analysis so far had the feature that $MPC_1^b = 1$, that is, borrowers consume all of their additional income. This feature is useful to illustrate our welfare results sharply, but it is rather extreme. We next analyze a version of our model in which borrowers' MPC can be flexibly parameterized. To keep the analysis simple, we assume $u(c) = \log c$ in this section so that we can calculate households' MPCs in closed form.

The main difference is that borrowers are now subject to heterogeneous shocks at date 1 that generate heterogeneity in their MPCs—and lower their MPCs as a group. In practice, there are many shocks that could create heterogeneity along these lines (e.g., income shocks). In our analysis, we find it convenient to introduce this heterogeneity through shocks to constraints. Specifically, all borrowers are identical at date 0 but they realize one of two types starting date 1. A fraction $\alpha \in [0, 1]$ of borrowers, denoted by type b_{con}, are subject to an exogenous borrowing constraint ϕ as before, and thus, they continue to have $MPC_1^{b_{con}} = 1$. The remaining fraction, denoted by type b_{unc}, are unconstrained at all dates, and thus they have a lower MPC. In particular, in view of the log utility, unconstrained borrowers—as well as

[13] This also follows the established practice of an existing academic literature that motivates macro-prudential policy based on alternative market imperfections (see our literature review on page 6 for a detailed list of references). For a more general discussion of the scope of macro-prudential policy, see for example Jeanne and Korinek (2014).

lenders—consume a small and constant fraction of the additional income they receive. To simplify the expressions, suppose also that all households have the same discount factor starting date 1 denoted by β (as before, borrowers have a lower discount factor at date 0, $\beta^b \leq \beta^l$). This implies:

$$MPC_1^l = MPC_1^{b_{unc}} = 1 - \beta. \tag{22}$$

Hence, the MPC of borrowers as a group is given by:

$$MPC_1^b \equiv \alpha + (1-\alpha)(1-\beta). \tag{23}$$

In particular, the parameter α enables us to calibrate the MPC differences between borrowers and lenders.

We simplify the analysis by assuming that borrowers are identical at date 0 and cannot trade assets whose payoffs are contingent on the type shocks they will receive at date 1. This ensures that each borrower enters date 1 with the same amount of outstanding debt d_1. To obtain slightly more general formulas, we also parameterize the relative mass of borrowers and lenders. Assume that the mass of lenders is given by ω, and that of borrowers by 1 so that $1/(1+\omega)$ denotes borrowers' share of the population. The baseline model is the special case with $\alpha = 1$ and $\omega = 1$.

As before, there is a threshold debt level $\bar{d}_1$, such that the equilibrium features a liquidity trap if and only if $d_1 > \bar{d}_1$. The analysis in Appendix A.2 further shows that

$$(1+\omega)\frac{\mathbf{d}e_1}{\mathbf{d}d_1} = -\frac{\alpha}{1-\frac{\alpha}{1+\omega}} = -\frac{MPC_1^b - MPC_1^l}{1-\overline{MPC}_1}, \tag{24}$$

where $\overline{MPC}_1 = \frac{MPC_1^b + \omega MPC_1^l}{1+\omega}$ denotes the average MPC across all households. Here, the left hand side illustrates the marginal effect of debt on total net demand, $e_1(1+\omega)$ (which takes into account the total size of the population). As before, greater debt induces a deeper recession. However, the strength of the effect now depends on the MPC differences between borrowers and lenders. Intuitively, greater debt influences aggregate demand by transferring wealth at date 1 from borrowers to lenders. This transfer affects demand more when there is a greater difference between borrowers' and lenders' MPCs. The effect is further exacerbated by the Keynesian income multiplier as captured by the denominator in (24).

We next characterize the planner's optimality condition as well as the optimal tax rate on borrowing for this case—the analogues of Eqs. (20) and (21). With some abuse of notation, we let $u'\left(c_1^b\right) = \alpha u'\left(c_1^{bcon}\right) + (1-\alpha)\, u'\left(c_1^{bcon}\right)$ denote borrowers' expected marginal utility before the realization of their type at date 1. The first order condition for the constrained planning problem stated in the appendix implies,

$$\frac{\beta^l u'\left(c_1^l\right)}{u'\left(c_0^l\right)} = \frac{\beta^b u'\left(c_1^b\right)}{u'\left(c_0^b\right)} - \left(\frac{\beta^b u'\left(c_1^b\right)}{u'\left(c_0^b\right)} + \omega \frac{\beta^l u'\left(c_1^l\right)}{u'\left(c_0^l\right)}\right) \frac{\mathbf{d}e_1}{\mathbf{d}d_1}, \tag{25}$$

for each $d_1 > \bar{d}_1$. Observe that the planner takes into account the negative effects of debt on all households' net incomes, as well as their welfare, as captured by the bracketed term. Combining Eqs. (21) and (25), and using a first order approximation (around $\tau_0^b \simeq 0$), we further obtain:

$$\tau_0^b \simeq -(1+\omega) \frac{\mathbf{d}e_1}{\mathbf{d}d_1} = \frac{MPC_1^b - MPC_1^l}{1 - \overline{MPC}_1}. \tag{26}$$

Thus, the optimal tax on borrowing is (to a first order) equal to the marginal effect of debt on total net demand. This is in turn equal to the MPC differences between borrowers and lenders, amplified by the Keynesian income multiplier.

Appendix A.2 generalizes these results to a setting with multiple (identifiable) groups of borrowers each of which might have different MPCs at date 1 (due to different α's). The analysis also accommodates multiple groups of lenders, some of which might have higher MPCs at date 1 (perhaps because they have relatively low assets and might become constrained with some probability). The optimal tax rate in (26) continues to apply for each group of borrowers or lenders, once we interpret group l in the formula as fully unconstrained households with $MPC_1^l = 1-\beta$ [see Eq. (A.21)]. However, the implementation with multiple groups features two differences relative to Corollary 2. First, the policies are non-anonymous in the sense that a particular tax rate τ_0^h applies only to group h households (as opposed to all households). Second, the tax rate applies to all debt choices by this group—as opposed to only positive debt issuance. In fact, a tax on negative debt issuance, $d_0^h < 0$, is in effect a subsidy for saving. The planner might use these subsidies to raise the saving by lenders with relatively high MPCs.

The empirical literature finds that the MPCs of households indeed differed greatly depending on their debt or asset position in the recent deleveraging episode. For example, Baker (2014) finds that the MPC out of income shocks for households in the highest decile of the debt-to-asset ratio distribution was about 57% (Figure 9), whereas the MPC of households without debt but positive net worth was about 26%, with an average MPC in the population of 30% (Table 4).[14] Our analysis suggests that the results from this literature can be used to guide optimal macroprudential policy. However, the formula in (26) assumes that the deleveraging episode occurs with probability one. This is useful for expositional simplicity, but would deliver unrealistically high tax rates. To address this, our next step is to introduce uncertainty into our framework.

5 Uncertainty about the deleveraging episode

Our analysis so far has focused on a special case in which the deleveraging episode is perfectly foreseen. This section extends the model to incorporate uncertainty about deleveraging. We first considers the case that financial markets are complete at date 0 so that households can trade insurance contracts contingent on the deleveraging episode. In this context, we establish our second main result that borrowers in a competitive equilibrium purchase too little insurance. We then consider the case in which financial markets are incomplete in the sense that households cannot trade contingent contracts, and generalize our excessive leverage result to this setting.

5.1 Deleveraging driven by insurable shocks

Consider the baseline setting described in Section 2, but suppose the economy is in one of two states $s \in \{H, L\}$ from date 1 onwards. The states differ in their debt limits. State L captures a "low leverage" state in which the economy experiences a financial shock and becomes subject to a permanent debt limit, $\phi_{t+1,L} \equiv \phi$ for each $t \geq 1$. State H in contrast captures the "high leverage" state in which households' debt choices remain unconstrained similar to date 0 of the earlier analysis, that is, $\phi_{t+1,H} = \infty$ for each $t \geq 1$. We use π_s^h to denote group h households' belief for state s and $E^h[\cdot]$ to denote their expectation operator over states. We assume $\pi_L^h > 0$ $\forall h$ so that the deleveraging episode is anticipated by all households.

We simplify the analysis by assuming that starting date 1, both types of households have the same discount factor denoted by β.[15] As before, borrowers have a lower discount factor at date 0 denoted by $\beta^b \leq \beta^l$. In addition, we also assume

[14] See also the survey by Jappelli and Pistaferri (2010) and recent papers by Mian et al. (2013) and Parker et al. (2013).

[15] This ensures that the equilibrium is non-degenerate in the high state H. Alternatively, we could impose a finite debt limit $\phi_{t+1,H} < \infty$.

borrowers are (weakly) more optimistic than lenders about the likelihood of the unconstrained state, $\pi_H^b \geq \pi_H^l$. Neither of these assumptions is necessary, but since impatience/myopia and excessive optimism were viewed as important contributing factors to many deleveraging crises, they enable us to obtain additional interesting results. We also replace the second part of Assumption (1) with the appropriate limit on d_0 for this case so that the interest rate constraint does not bind at date 0.

At date 0, households are allowed to trade two types of securities. First, as before, they choose their debt (or asset) level d_{t+1}^h for the next period. The debt is non-contingent in the sense that it promises the same payment $1 + r_{t+1}$ (per unit) in each state s, where r_{t+1} denotes the safe real interest rate as before. Second, households can also hold an Arrow-Debreu security that pays 1 unit of the consumption good in state L and nothing in the other state. We refer to this asset as an *insurance contract*, and denote household's position in this asset with m_L^h and the price of the asset with q_L. Households' budget constraint can be written (in net variables) as:

$$\begin{aligned} c_0^h &= e_0 - d_0^h + \frac{d_1^h}{1+r_1} - m_L^h q_L^h, \\ \text{and } c_{1,s}^h &= e_{1,s} - d_{1,s}^h + \frac{d_{2,s}^h}{1+r_1}, \text{ where } \left\{ \begin{array}{c} d_{1,L}^h \equiv d_1^h - m_L^h \\ d_{1,H}^h \equiv d_1^h \end{array} \right. . \end{aligned}$$

Here, $d_{1,s}^h$ denotes households' *effective debt level* in state s. Note that the two securities complete the market in the sense that they enable the households to freely choose their effective debt (or asset) levels. Given these changes, the optimization problem of households and the definition of equilibrium generalize to uncertainty in a straightforward way. We also let $d_{1,s} \equiv d_{1,s}^b$ denote the effective aggregate debt level in state s and $m_L \equiv m_L^b$ denote borrowers' aggregate insurance purchase.

The equilibrium in state L is the same as described as before. In particular, the interest rate is zero and there is a demand-driven recession as long as the effective debt level exceeds a threshold, $d_{1,L} > \bar{d}_1$. The equilibrium in state H jumps immediately to a steady-state with interest rate $1 + r_{t+1} = 1/\beta > 0$ and consumption $c_{t,H}^h = e^* - (1-\beta)\, d_{1,H}^h \;\forall t \geq 1$.

The main difference concerns households' date 0 choices. In this case, households' optimal debt choice implies Euler equations as before,

$$\frac{1}{1+r_1} = \frac{\beta^l E^l \left[u'\left(c_1^l\right)\right]}{u'\left(c_0^l\right)} = \frac{\beta^b E^b \left[u'\left(c_1^b\right)\right]}{u'\left(c_0^b\right)}, \tag{27}$$

and their optimal insurance choice implies *full insurance conditions* for state L,

$$q_{1,L} = \frac{\beta^l \pi_L^l u'\left(c_{1,L}^l\right)}{u'\left(c_0^l\right)} = \frac{\beta^b \pi_L^b u'\left(c_{1,L}^b\right)}{u'\left(c_0^b\right)}. \tag{28}$$

We next describe under which conditions households choose a sufficiently high debt level for state L to trigger a recession, $d_{1,L} > \bar{d}_1$:

Proposition 3. *There is a deleveraging-induced recession in state L of date 1 if the borrower is either (i) sufficiently impatient or (ii) sufficiently indebted or (iii) sufficiently optimistic at date 0. Specifically, for any two of the parameters $\left(\beta^b, d_0, \pi_L^b\right)$, we can determine a threshold for the third parameter such that $d_{1,L} > \bar{d}_1$ if the threshold is crossed, i.e. if $\beta^b < \bar{\beta}^b\left(d_0, \pi_L^b\right)$ or $d_0 > \bar{d}_0\left(\beta^b, \pi_L^b\right)$ or $\pi_L^b < \bar{\pi}_L^b\left(\beta^b, d_0\right)$.*

The thresholds are characterized in more detail in Appendix A.1. The first two cases are analogous to the cases in Proposition 1: if borrowers have a strong motive to carry debt into date 1, they also choose to hold a large level of effective debt in state L, even though this triggers a recession. The last case identifies a new factor that could exacerbate this outcome. If borrowers assign a sufficiently low probability to state L, relative to lenders, then they also naturally choose to hold a large level of effective debt in state L. In each scenario, $d_{1,L} > \bar{d}_1$ and there is a recession in state L of date 1.

To analyze welfare, consider a planner who can choose households' allocations at date 0 and control their effective debt levels at date 1 (via the simple policies we will describe below), but leaves the remaining allocations to the market. The constrained planning problem can be written as:

$$\max_{\left(\left(c_0^h, n_0^h\right)_h, d_{1,H}, d_{1,L}\right)} \sum_h \gamma^h \left[u\left(c_0^h\right) + \beta^h \sum_s \pi_s^h V_s^h\left(d_{1,s}^h, d_{1,s}\right)\right] \tag{29}$$
$$\text{such that } d_{1,s} = d_{1,s}^b = -d_{1,s}^l \text{ for each } s, \text{ and } \sum_h c_0^h = \sum_h \left[n_0^h - v\left(n_0^h\right)\right].$$

Our next result characterizes the solution to this problem over the relevant range.

Proposition 4 (Optimal Insurance)**.** *An allocation $\left(\left(c_0^h, n_0^h\right)_h, d_{1,H}, d_{1,L}\right)$, with $d_{1,L} \geq \phi$ and $u'\left(c_0^l\right) \geq \beta^l E^l\left[u'\left(c_1^l\right)\right]$, is constrained efficient if and only if output at date 0 is efficient, i.e., $e_0 = e^*$; households' full insurance condition for state H holds, $\frac{\beta^l \pi_H^l u'\left(c_{1,H}^l\right)}{u'\left(c_0^l\right)} = \frac{\beta^b \pi_H^b u'\left(c_{1,H}^b\right)}{u'\left(c_0^b\right)}$; and the remaining consumption and leverage allocations satisfy one of the following:*

(i) $d_{1,L} \leq \bar{d}_1$ *and the full insurance conditions (28) also hold for state* L*,*
(ii) $d_{1,L} = \bar{d}_1$ *and the following inequality holds for state* L*:*

$$\frac{\beta^l \pi^l_L u'\left(c^l_{1,L}\right)}{u'\left(c^l_0\right)} > \frac{\beta^b \pi^b_L u'\left(c^b_{1,L}\right)}{u'\left(c^b_0\right)}. \tag{30}$$

The second part illustrates our main result with uncertainty. The planner limits the effective debt level in the deleveraging episode, $d_{1,L} = \bar{d}_1$, and distorts households' insurance conditions according to (30). As this inequality illustrates, borrowers would like to reduce their insurance purchases (which would raise their effective debt in state L) so as to consume more in state 0 and less in state L, but they are prevented from doing so by the planner. In particular, a competitive equilibrium with $d_{1,L} > \bar{d}_1$ is constrained inefficient, as we formalize next.

Corollary 3 (Underinsurance)**.** *The competitive equilibrium allocation* $\left((c^{h,eq}_0, n^{h,eq}_0)_h, d^{eq}_{1,H}, d^{eq}_{1,L}\right)$ *in Proposition 3 is constrained inefficient, and it is Pareto dominated by the constrained efficient allocation* $\left(c^h_0, n^h_0\right)_h = (c^{h,eq}_0, n^{h,eq}_0)_h, d_{1,H} = d^{eq}_{1,H}$ *and* $d^{eq}_{1,L} = \bar{d}_1$*.*

This result identifies a distinct type of inefficiency in our setting: borrowers in a competitive equilibrium buy too little insurance with respect to aggregate deleveraging episodes. Intuitively, they do not take into account the positive aggregate demand externalities their insurance purchases would bring about. Therefore, they end up with financial portfolios that are too risky from a social point of view.

We next show that the constrained efficient allocations can be implemented with macroprudential insurance policies. First, suppose the planner can require households' effective debt level in state L to be bounded from above, that is, $d^h_{1,L} \leq \bar{d}_1$ for each h. This is equivalent to setting a *minimum insurance requirement* that depends on households' total debt, $m^h_L \geq d^h_1 - \bar{d}_1$. Note the planner is setting a tighter requirement for more indebted households. Second, suppose the planner can also set a linear subsidy (or tax) on borrowers' insurance positions, m^b_L. Specifically, m^b_L units of the insurance contract costs the borrowers $m^b_L q_L \left(1 - \zeta^b_0\right)$ units of the consumption good. Note that this policy corresponds to a subsidy to borrowers when they purchase insurance $m^b_L > 0$, but it would correspond to a tax if they chose to sell insurance $m^b_L < 0$. The policy does not apply to lenders (and thus, is not anonymous), who continue to receive or pay q_L per unit of the insurance contract.[16] The total cost

[16] The planner needs non-anonymous policies in this case because, in view of belief disagreements,

of the subsidy is $m_L \zeta_0^b$, which is financed by lump-sum taxes on all households. As before, the planner can also combine these policies with a transfer of wealth T_0^b from lenders to borrowers.

Corollary 4 (Implementing the Optimal Insurance)**.** *The constrained efficient allocations characterized in Proposition 4 can be implemented alternatively with:*

(i) the minimum insurance requirement, $m_L^h \geq d_1^h - \bar{d}_1$ for each h, or

(ii) insurance subsidies to borrowers, $\zeta_0^b > 0$, that satisfy:

$$\frac{\beta^l \pi_L^l u'\left(c_1^l\right)}{u'\left(c_0^l\right)} = \frac{\beta^b \pi_L^b u'\left(c_1^b\right)}{u'\left(c_0^b\right)} \frac{1}{1-\zeta_0^b}, \tag{31}$$

combined with an appropriate ex-ante transfer T_0^b for each case.

The insurance requirement directly restricts borrowers' outstanding debt in state L. The subsidy policy brings about the same outcome by lowering the net-of-tax insurance price that borrowers face relative to lenders. We can also quantify the optimal subsidy in our setting after modifying the model as in Section 4.3 so as to flexibly parameterize households' MPCs. Appendix A.2 specifies the details and obtains the following analogue of Eq. (26):

$$\zeta_0^b \simeq -\left(1+\omega\right) \frac{\mathbf{d}e_{1,L}}{\mathbf{d}d_{1,L}} = \frac{MPC_1^b - MPC_1^l}{1-\overline{MPC}_1}. \tag{32}$$

That is, the optimal subsidy rate—just like the optimal tax rate—is equal to (up to a first order) the marginal effect of lowering borrowers' debt (in state L) on total net demand, $(1+\omega)e_{1,L}$. This in turn is equal to the MPC differences between borrowers and lenders. Like the optimal tax rate, this formula also generalizes to a setting with multiple groups of borrowers or lenders [see Eq. (A.23)].

Our model has many stylized features, departing from which would naturally affect the optimal subsidy (as well as tax) formulas.[17] Nonetheless, we view the formula in (32) as providing a useful benchmark for understanding the order of magnitude

borrowers might choose to sell insurance as opposed to buying. If this happens, subsidizing insurance purchases anonymously would create the opposite of the intended effect. In the special case with common beliefs, $\pi_0^l = \pi_0^b$, the equilibrium features insurance purchases by borrowers, $m_L^b > 0$, and the anonymous policy also works.

[17]For instance, considering a different utility function than the GHH form, $u\left(c - v\left(n\right)\right)$, would typically affect the optimal subsidy and tax formulas. Appendix B.4 analyzes separable preferences, $u\left(c\right) - v\left(n\right)$, and shows that the optimal tax rate in that case also depends on the labor wedge—which measures the welfare benefits of raising aggregate output net of the increased costs of labor supply [see Eqs. (B.14) and (B.15)]. For another example, introducing investment into the model

for reasonable subsidy policies in practice. Using the MPC estimates of borrowers and lenders from Baker (2014) that we cited at the end of Section 4, a back-of-the-envelope calculation suggests an insurance subsidy in the order of 44% [= (0.57-0.26)/(1 - 0.30)] for the most leveraged decile of borrowers.[18]

An important financial shock in practice is an economy-wide (or widespread) decline in house prices, which can considerably tighten many homeowners' borrowing constraints and induce a demand-driven recession. In this context, Proposition 4 provides a rationale for policies that reduce mortgage borrowers' exposures to house prices. Corollary 4 and Eq. (32) illustrate how this can be implemented by subsidizing a type of *home equity insurance* that pays homeowners when there is a severe and economy-wide downturn in house prices.[19]

Shiller and Weiss (1999) proposed home equity insurance along these lines to protect homeowners against declines in housing prices, but demand for such insurance has been muted (see Shiller, 2003). In the recent housing boom, one of the reasons why households wanted to expose themselves to house price risk was arguably their optimism about house prices increases (see Case, Shiller, and Thompson, 2012). This optimism is also one of the factors that we capture in Proposition 3. However, our model emphasizes that there are in addition uninternalized social benefits to home equity insurance for highly leveraged borrowers since they create aggregate demand externalities. The resulting underinsurance is socially inefficient and provides a rationale for insurance requirements or subsidies, even if policymakers respect households' different beliefs.

would affect, among other things, the Keynesian multiplier in the denominator of the formulas in (26) and (32).

[18]This number (44%) might sound large, but note that this is a subsidy only on the insurance bill of borrowers for severe deleveraging episodes, which is likely to be much smaller than their loan balances.

[19]It is important to emphasize that our model does not support subsidizing insurance to all types of home equity insurance. For instance, insurance with respect to idiosyncratic events that lower the value of a small number of houses (such as a fire or a local flood) are not supported by our policies, since these events are unlikely to influence the aggregate demand. In contrast, our analysis supports subsidizing insurance with respect to aggregate shocks to house prices, which could induce widespread deleveraging and influence the aggregate demand. These insurance markets are arguably also resilient to moral hazard or adverse selection, since individual insurance buyers are unlikely to influence the probability of aggregate events or to have private information about the probability of such events.

5.2 Deleveraging driven by uninsurable shocks

While certain financial shocks that trigger deleveraging seem possible to insure against (at least in principle), other shocks can be much more difficult to contract. Consider, for example, the recent subprime financial crisis that put many financial institutions into distress, which arguably lowered credit to households even after controlling for their collateral values (see Mondragon (2015) for empirical evidence). Caballero and Simsek (2013) describe several reasons that could have made it difficult or costly to purchase ex-ante insurance protection against this event.[20] Perhaps for these (or other) reasons, the recent literature on financial frictions typically assume that insurance markets are missing for aggregate financial shocks (see Brunnermeier and Sannikov (2014) and the references therein).

What becomes of our welfare analysis when the underlying financial shock—and therefore, the deleveraging episode—is uninsurable? To investigate this question, consider the model with uncertainty with the only difference that households cannot trade insurance contracts, so that, $m_L^h = 0$ for each h. To obtain slightly more general and quantifiable formulas, suppose also that the model is modified as in Section 4.3 (see Appendix A.2 for details). The equilibrium at date 0 is characterized by households' Euler equations analogous to Eq. (27). Under appropriate conditions, there is a recession in state L as in Proposition 4.

To analyze constrained efficiency, suppose the planner is also subject to the same market incompleteness, so that she is constrained to choose $d_1 = d_{1,H} = d_{1,L}$. Appendix A.2 shows that the constrained efficient allocations in this case satisfy:[21]

$$\frac{\beta^l E^l\left[u'\left(c_1^l\right)\right]}{u'\left(c_0^l\right)} = \frac{\beta^b E^b\left[u'\left(c_1^b\right)\right]}{u'\left(c_0^b\right)} - \left(\pi^b \frac{\beta^b u'\left(c_{1,L}^b\right)}{u'\left(c_0^b\right)} + \pi^l \omega \frac{\beta^l u'\left(c_{1,L}^l\right)}{u'\left(c_0^l\right)}\right) \frac{\mathbf{d} e_{1,L}}{\mathbf{d} d_1}, \qquad (33)$$

where $\frac{\mathbf{d}e_{1,L}}{\mathbf{d}d_1} < 0$ as before (see Eq. (24)). Thus, the competitive equilibrium features excessive leverage also in this case. However, unlike in Section 4.3, the size of

[20] First, while the problems in the subprime market were anticipated, the exact location or the magnitude of the losses remained not understood until well into 2008. Given that the event was difficult to describe, it was also arguably difficult (or very costly) to insure against. Second, the shock was systemic and the potential insurance sellers (which are left out of our analysis) also became distressed, which would have further increased the costs of insurance.

[21] In general, this type of environment might also feature pecuniary externalities that need not net out because markets are incomplete across states H and L (cf. Lemma 2). In our setting, these pecuniary externalities are mute because the interest rate is constant in both states (over the relevant range), that is, $1 + r_H = 1/\beta$ and $1 + r_L = 1$. Consequently, the constrained optimailty condition features only aggregate demand externalities.

the required intervention also depends on households' perceived probabilities for the deleveraging episode, π^b and π^l.

In this case, there is no simple first order approximation to the optimal tax rate on borrowing (since households' marginal utilities are not necessarily equated at the no-tax benchmark). For a back-of-the-envelope calculation, consider the special case in which households agree about the probability of the deleveraging episode, $\pi \equiv \pi^b = \pi^l$. Suppose also that the no-tax allocations roughly satisfy $u'\left(c_{1,L}^h\right) \simeq u'\left(c_{1,H}^h\right)$ for each h. With these simplifications, Eq. (33) implies (up to a first order):

$$\tau^b \simeq -\pi\left(1+\omega\right)\frac{\mathbf{d}e_{1,L}}{\mathbf{d}d_1} = \pi \cdot \frac{MPC_1^b - MPC_1^l}{1-\overline{MPC}_1}. \tag{34}$$

In particular, the optimal tax rate is roughly approximated as the tax rate without uncertainty [cf. Eq. (26)] multiplied by the probability of the deleveraging episode. As before, although the formula depends on special features of our model, we view it as helpful in understanding the order of magnitude for reasonable tax policies in practice. Using our earlier MPC estimates (Baker, 2014) and a probability of once every thirty years for a deleveraging crisis (e.g. Reinhart and Rogoff, 2009), Eq. (34) would suggest a tax rate in the order of 1.5% on noncontingent debt for the most leveraged decile of borrowers.

Intuitively, macroprudential policies that restrict debt provide blanket protection with respect to all deleveraging episodes, including those driven by uninsurable ones. These policies bring benefits by raising aggregate demand when the episode is realized. However, they also generate costs by distorting households' consumption in many other future states without deleveraging. Consequently, their desirability depends on the probability of deleveraging. This result also illustrates how the optimal macroprudential regulation is likely to be time (as well as context) dependent. Policies that are optimal for a particular time and environment might cease to be optimal in the future, e.g., if deleveraging becomes less likely or if other unmodeled considerations become relevant. This highlights the importance of continuous monitoring and analysis for optimal macroprudential regulation.

Let us summarize our findings on optimal macroprudential policy interventions to correct aggregate demand externalities: Sections 4.2 and 4.3 showed that it is optimal to tax borrowing to protect an economy against a foreseen episode of deleveraging. Section 5.2 adjusted the resulting tax rate by the probability of the deleveraging episode. The common theme is that the policymaker induces households to internalize the social cost of carrying too much debt when the economy experiences deleveraging.

Section 5.1 showed that when insurance markets are available, it is more efficient to focus the planner's intervention solely on the state(s) of nature in which deleveraging occurs. The planner leaves it to private households to choose the total level of debt but induces them to buy insurance against the deleveraging episode.

6 Preventive monetary policies

The analysis so far has focused on macroprudential policies, i.e., interventions in financial markets. A natural question is whether preventive monetary policies could also be desirable to mitigate the inefficiencies in this environment. In this section, we analyze respectively the effect of changing the inflation target and adopting a contractionary monetary policy.

6.1 Changing the inflation target

Blanchard, Dell'Ariccia and Mauro (BDM, 2010), among others, emphasized that a higher inflation target could be useful to avoid or mitigate the liquidity trap. We illustrate this point using the version of our model with intermediately sticky prices and an inflation targeting Taylor rule, developed in Appendix B.1. There, the Taylor rule ensures that the aggregate price inflation between dates 1 and 2 is equal to the inflation target, that is, $P_2/P_1 = \Pi$ where Π is the gross inflation target. Combining this with (3), the real interest rate is bounded from below, that is, $1 + r_2 \geq 1/\Pi$. It follows that raising the inflation target Π relaxes the bound on the real rate. Consequently, a greater level of leverage is necessary to plunge the economy into a liquidity trap (see Appendix B.1). Hence, raising the inflation target reduces the incidence of liquidity traps, consistent with BDM (2010), as well as the incidence of aggregate demand externalities. These welfare benefits should be weighed against the various costs of higher steady-state inflation.

6.2 Contractionary monetary policy

It has also been discussed that interest rate policy could be used as a preventive measure against financial crises. In fact, a number of economists have argued that the US Federal Reserve should have raised interest rates in the mid-2000s in order to lean against the housing bubble or to reduce leverage (see Woodford (2012) and Rajan (2010) for detailed discussions). We next investigate the effect of contractionary policy at date 0 on household leverage.

To this end, consider the baseline setting with a single type of borrower and no uncertainty. Suppose the conditions in Proposition 1 apply so that there is a liquidity trap at date 1. Suppose the monetary authority sets $r_1 > r_1^*$ at date 0, and follows the rule in (10) thereafter. In this case, the equilibrium at date 0 features a policy-induced recession, that is, households' net income falls to $e_0 < e^*$. Moreover, households' Euler equations are now given by

$$\frac{1}{1+r_1} = \frac{\beta^l u'\left(e_1 + (d_1 - \phi)\right)}{u'\left(e_0 + d_0 - \frac{d_1}{1+r_1}\right)} = \frac{\beta^b u'\left(e_1 - (d_1 - \phi)\right)}{u'\left(e_0 - \left(d_0 - \frac{d_1}{1+r_1}\right)\right)}, \tag{35}$$

where $e_1 = \bar{c}_1^l - (d_1 - \phi) < e^*$ as in (15). This describes two equations in two unknowns, $e_0(r_1), d_1(r_1)$, which can be solved as a function of the policy rate r_1. Our next result characterizes the comparative statics with respect to r_1.

Proposition 5 (Contractionary Monetary Policy)**.** *Consider the equilibrium described above with a liquidity trap at date* 1 *and the interest rate* $r_1 > r_1^*$. *Suppose* $-u''(x)/u(x)$ *is a weakly decreasing function of* x. *Suppose also that* d_0 *is sufficiently large so that* $d_0 - \frac{d_1(r_1)}{1+r_1} > 0$. *Then,* $e_0'(r_1) < 0$ *and* $d_1'(r_1) > 0$, *that is: increasing the interest rate* r_1 *decreases the current net income and increases the outstanding debt level* d_1.

The proposition considers cases in which the utility function lies in the decreasing absolute risk aversion family—which encompasses the commonly used constant elasticity case—and lenders' initial assets are sufficiently large so that their consumption exceeds borrowers' consumption (see (35)). As expected, raising the interest rate in the run-up to a deleveraging episode creates a recession. Perhaps surprisingly, under symmetry assumptions, raising the interest rate in our setting also *increases* the equilibrium leverage. This in turn leads to a more severe recession at date 1.

To understand this result, suppose $u(c) = \log c$ and $\phi = 0$. In this case, borrowers' and lenders' optimal debt choices have closed form solutions, conditional on the income levels e_0 and e_1, given by

$$\begin{aligned} d_1^b &= \frac{1}{1+\beta^b}\left(e_1 - \beta^b (1+r_1)(e_0 - d_0)\right) \\ d_1^l &= \frac{1}{1+\beta^l}\left(e_1 - \beta^l (1+r_1)(e_0 + d_0)\right). \end{aligned} \tag{36}$$

In particular, keeping e_0 and e_1 constant, a higher r_1 reduces both d_1^b and d_1^l. Intuitively, the substitution effect induces borrowers to borrow less but also induces lenders to save more. This creates an excess demand in the asset market (that is, $d_1^b + d_1^l$ falls below 0)—or equivalently, a shortage of demand in the goods market. To equilibrate markets, output falls and households' net income e_0 declines. As this happens, both d_1^b and d_1^l increases: that is, borrowers borrow more and lenders save less so as to smooth their consumption. In our model, these effects are roughly balanced across borrowers and lenders since all households share the same elasticity of intertemporal substitution. In fact, if d_0 were equal to 0, the reduction in e_0 would be (with log utility) just enough to counter the initial effect and the equilibrium debt level $d_1 = d_1^b = -d_1^l$ would remain unchanged (see (36)). When d_0 is sufficiently large, higher r_1 creates an additional wealth transfer from borrowers to lenders. This increases borrowers' debt d_1^b further—while increasing lenders' assets—generating a higher equilibrium debt level $d_1 = d_1^b$. The proof in the appendix uses more subtle arguments to establish the result more generally.

Hence, the conventional wisdom—that raising the interest rate decreases leverage—fails in view of two general equilibrium effects on borrowers' income and wealth. First, the higher interest rate creates a temporary recession, which reduces borrowers' current income and induces them to take on greater debt. Second, the higher interest rate also transfers wealth from borrowers to lenders, which further increases borrowers' debt. The combination of these two effects can dominate the partial equilibrium effect of the higher interest rate on borrowers, leading to greater debt in equilibrium.

We could construct variants of our model in which raising the interest rate decreases the outstanding leverage, d_1. For instance, if borrowers' intertemporal substitution is more elastic than lenders', as in Cúrdia and Woodford (2009), then the equilibrium debt level might decrease due to a stronger substitution effect for borrowers. However, even in these cases, the interest rate policy would not be the optimal instrument to deal with the excessive leverage problem. The following proposition establishes this point by characterizing the jointly optimal monetary and macroprudential policies at date 0.

Proposition 6 (Jointly Optimal Monetary and Macroprudential Policy)**.** *Consider the baseline model with the only difference that borrowers and lenders have heterogeneous utility functions at (only) date* 0, $u_0^b(\cdot)$ *and* $u_0^l(\cdot)$ *(so as to allow for richer effects of monetary policy on leverage). Suppose the planner chooses the interest rate* r_1 *at date* 0*, in addition to setting the macroprudential policies described in Section 4. It is optimal for this planner to set* $r_1 = r_1^*$ *and implement* $e_0 = e^*$*.*

That is, once macroprudential policies are in place, it is optimal for the monetary policy to pursue its myopic output stabilization goal. Intuitively, the constrained efficient allocations characterized in Proposition 2 feature the frictionless output level at date 0. Moreover, those constrained efficient allocations can be implemented with macroprudential policies alone. It follows that raising the interest rate is not desirable, because it triggers an inefficient recession at date 0 without providing any benefits over and above macroprudential policies.

These results illustrate that the interest rate policy is not the right tool to deal with the excessive leverage problem. The problem is one of inefficient distribution of financial wealth between borrowers and lenders during the liquidity trap episode. Consequently, the constrained efficient allocations require creating a wedge between borrowers' and lenders' interest rates [see Corollary 2 and Eq. (21)]. In contrast, monetary policy sets a different "intertemporal" wedge that affects both borrowers' and lenders' interest rates. Given that monetary policy targets "the wrong wedge," it could at best be viewed as a crude solution for dealing with excessive leverage. In contrast, macroprudential policies, e.g., debt limits or insurance subsidies, optimally internalize aggregate demand externalities created by leverage.

It is important to emphasize that contractionary monetary policy could well be desirable for reasons outside the scope of our model. For instance, if macroprudential policies are not available, then raising the interest rate might be useful to mitigate inefficient investment booms and fire-sale externalities as in Lorenzoni (2008) or Stein (2012). A higher interest rate might also be useful to lean against asset price bubbles, e.g., by discouraging the "search for yield" phenomenon discussed in Rajan (2010). Our point is that contractionary monetary policy is not the ideal instrument to reduce household leverage, and in fact, might have the unintended consequence of raising leverage.

7 Aggregate demand and fire-sale externalities

In this section we endogenize the debt limit faced by borrowers by assuming that debt is collateralized by a financial asset, creating the potential for fire-sale effects. This introduces a new feedback loop into the economy: first, a decline in asset prices reduces the borrowing capacity of households and forces them to delever, giving rise to financial amplification; secondly, in a liquidity trap, deleveraging leads to a demand-induced decline in output that triggers Keynesian multiplier effects. The two feedback effects mutually reinforce each other. As a result, a recession involving deleveraging and fire-sale effects of collateral assets may be particularly severe.[22]

We modify our earlier setup by assuming that borrowers hold one unit $a_t = 1$ of a tree from which they obtain a dividend y_t every date. For simplicity, we assume that the tree only pays dividends if it is owned by borrowers so the tree cannot be sold to lenders. The tree trades among borrowers at a market price of p_t. We follow Jeanne and Korinek (2010b) in assuming that borrowers are subject to a moral hazard problem and have the option to abscond with their loans after the market for loans has closed. In order to alleviate the moral hazard problem, they pledge their trees as collateral to lenders. When a borrower absconds with her loan, lenders can detect this and can seize up to a fraction $\phi_{t+1} < 1$ of the collateral and sell it to other borrowers. The borrowing constraint is therefore endogenous and given by:

$$d_{t+1}/\left(1 + r_{t+1}\right) \leq \phi_{t+1} a_{t+1} p_t.$$

Similar to earlier, we assume $\phi_1 = 1$ and $\phi_{t+1} = \phi < 1$ for each $t \geq 1$. Deleveraging may now be driven by two separate forces: a decline in the pledgeability parameter, ϕ_t, and a decline in the price of the collateral asset. We will see shortly that declines in ϕ_t are generally amplified by asset price declines.

In the following, we make two simplifying assumptions. First, starting date $t = 2$, we assume that the output from the tree is a constant y and there are no further shocks. Second, we let the discount factors of the two households $\beta^b = \beta^l = \beta$. Together, these two assumptions imply that the economy will be in a steady state starting date 2 in which debt is constant at $d_t = d_2$ and the asset price and consumption satisfy $p_t = \frac{\beta}{1-\beta} y$, $c_t^b = y + e^* - (1-\beta) d_2$, $c_t^l = e^* + (1-\beta) d_2$ for $t \geq 2$ respectively.

We next consider the equilibrium at date 1 at which the asset's dividend is given by some $y_1 \leq y$. As before, if the debt level is sufficiently large, that is, $d_1 > \bar{d}_1$ for some threshold $\bar{d}_1$, then the economy is in a liquidity trap. In particular, borrowers are constrained, $d_2 = \phi p_1$, the interest rate is at zero, $r_2 = 0$, and output is below its efficient level, $e_1 < e^*$. Moreover, the equilibrium is determined lenders' Euler

[22] The interaction between asset fire sales and aggregate demand has also been studied in Carlstrom and Fuerst (1997), Bernanke et al. (1999) and Iacoviello (2004). In these papers, monetary policy can mitigate the feedback effects resulting from tightening borrowing constraints. We consider the possibility of a lower bound on interest rates that prevents this, and we add a normative dimension focused on debt market policies. There is also a related literature in open economy macroeconomics that analyzes how price-dependent financial constraints capture the dynamics of sudden stops in capital inflows, and how this may interact with monetary policy. See for instance Krugman (1999), Aghion et al. (2001, 2004), and Aguiar and Gopinath (2005).

equation at the zero interest rate:

$$u'\left(e_1 + d_1 - \phi p_1\right) = \beta u'\left(e^* + (1-\beta)\,\phi p_1\right). \tag{37}$$

The difference is that the asset price also enters this equation since higher prices increase the endogenous debt limit, which influences aggregate demand and output. The asset price is in turn characterized by:

$$\begin{aligned} p_1 &= MRS\left(e_1, p_1\right) \cdot p_2 = \frac{u'\left(c_2^b\right)}{(1-\phi)\,u'\left(c_1^b\right) + \phi\beta u'\left(c_2^b\right)} \cdot \frac{\beta y}{1-\beta}, \\ &\text{where } \left\{ \begin{array}{c} c_2^b = e^* + y - (1-\beta)\,\phi p_1 \\ c_1^b = e_1 + y_1 - d_1 + \phi p_1 \end{array} \right. . \end{aligned} \tag{38}$$

This captures that today's asset price is tomorrow's price $p_2 = \frac{\beta y}{1-\beta}$ discounted by the MRS applicable to asset purchases, which in turn reflects that a fraction ϕ of the asset can be purchased with borrowed funds. Since the extent of deleveraging at date 1 is endogenous to p_1, the MRS is itself a function of the asset price p_1. For the implicit asset price equation (38) to have a unique and well-defined solution, it is necessary that the slope of the left-hand side is higher than the slope of the right-hand side, i.e. $p_2 \cdot \partial MRS/\partial p_1 < 1$. (The condition is characterized in terms of fundamental parameters in the appendix.) We also observe that $\partial MRS/\partial e_1 > 0$ as higher income today makes borrowers more willing to buy assets. Therefore the equilibrium asset price defined by the equation is increasing in current income, $dp_1/de_1 > 0$. Furthermore, the asset price is increasing in the exogenous collateral limit, ϕ, which can be understood from a collateral value channel: A higher ϕ implies the asset is more useful to relax the borrowing constraint, which raises its price.

The equilibrium is characterized by two equations, (37) and (38), in two unknowns (e_1, p_1). The first equation describes an increasing relation, $e_1^{AD}\left(p_1\right)$, that represents the aggregate demand effects of asset prices. Intuitively, a higher price raises the endogenous debt level, which in turn raises aggregate demand and output. The second equation describes the consumer's asset pricing relationship $e_1^{AP}\left(p_1\right)$, i.e. it captures the level of income required to support a given asset price. It is also increasing under our earlier assumption on the MRS. Intuitively, supporting a higher asset price requires higher consumption and therefore a higher net income, e_1. Any intersection of these two curves, that also satisfies $\partial e_1^{AP}/\partial p_1 > \partial e_1^{AD}/\partial p_1$, is a stable equilibrium.

To analyze welfare, consider the externalities from leverage, $\frac{\mathbf{d}V^h}{\mathbf{d}d_1}$, which can now be written as:

$$
\begin{aligned}
\frac{\mathbf{d}V^l}{\mathbf{d}d_1} &= u'\left(c_1^l\right)\frac{\mathbf{d}e_1}{\mathbf{d}d_1}, \\
\frac{\mathbf{d}V^b}{\mathbf{d}d_1} &= u'\left(c_1^b\right)\frac{\mathbf{d}e_1}{\mathbf{d}d_1} + \phi\frac{\mathbf{d}p_1}{\mathbf{d}d_1}\left[u'\left(c_1^b\right) - \beta u'\left(c_2^b\right)\right],
\end{aligned}
$$

where $\frac{\mathbf{d}e_1}{\mathbf{d}d_1}$ and $\frac{\mathbf{d}p_1}{\mathbf{d}d_1}$ are jointly obtained from expressions (37) and (38) and are both negative under the assumptions made earlier. Note that the expression for both types of households features aggregate demand externalities. The expression for borrowers features in addition fire sale externalities. Intuitively, a higher debt level lowers borrowers' consumption, which in turn lowers the asset price. The low price in turn tightens borrowing constraints and leads to further declines in consumption and asset prices and so forth. This feedback mechanism occurs in addition to the Keynesian multiplier that we discussed before. As a result, aggregate demand externalities and fire sale externalities reinforce each other.

It follows that endogenizing the financial constraint as a function of asset prices reinforces the problems of excessive leverage and underinsurance through two channels. First, it introduces fire-sale externalities that operate on borrowers' welfare in the same direction as aggregate demand externalities. Second, it exacerbates aggregate demand externalities by tightening borrowing constraints further. The latter effect also illustrates an interesting mechanism through which asset price declines hurt all households in the economy via aggregate demand effects, even if they do not hold financial assets. In this model, lenders do not hold the asset, but they are nonetheless hurt by the price decline because it leads to more deleveraging and magnifies the recession.

8 Conclusion

When borrowers are forced to delever, the interest rate might fail to decline sufficiently to clear the goods market, plunging the economy into a liquidity trap. This paper analyzed the role of preventive policies in the run-up to such episodes. We established that the competitive equilibrium allocations feature excessive leverage and underinsurance in view of aggregate demand externalities. A planner can improve welfare and implement constrained efficient allocations by using macroprudential policies that restrict debt and incentivize borrowers' insurance. We also showed that optimal borrowing taxes and insurance subsidies depend on, among other things, the differences in the MPC out of liquid wealth between borrowers and lenders.

We showed that contractionary monetary policy that raises the interest rate cannot implement the constrained efficient allocations in this setting. Moreover, due to general equilibrium effects, this policy can have the unintended consequence of increasing household leverage and exacerbating aggregate demand externalities. That said, a contractionary monetary policy could well be desirable for reasons outside our model. We leave a more complete analysis of preventive monetary policies for future work.

Although we focus on consumption and household leverage, our mechanism also has implications for investment and firms' leverage. Similar to households, firms feature a great deal of heterogeneity in their propensities to invest out of liquidity. Moreover, although there is no consensus, firms that are more financially constrained seem to have greater propensity to invest (see, for instance, Rauh, 2006) especially during a financial crisis (see Campello, Graham, Harvey, 2010). Hence, transferring ex-post wealth from borrowing firms to "lending" firms (those with large holdings of cash) is likely to decrease investment and aggregate demand. Our main results then suggests that firms will also borrow too much, and purchase too little insurance, in the run-up to deleveraging episodes that coincide with a liquidity trap. Just like with households, these inefficiencies can be corrected with macroprudential policies such as debt limits and capital/insurance requirements.

Many macroprudential policies in practice concern banks (or financial institutions) that intermediate funds between ultimate lenders and borrowers. Our analysis can also be extended to provide a justification for some of these policies. A large literature in corporate finance has emphasized that banks' net worth affects credit supply, which in turn affects consumption or investment by credit constrained borrowers. In fact, in some theoretical benchmarks, banks' net worth is interchangeable with borrowers' net worth (see, for instance, Holmstrom and Tirole (1997) or Brunnermeier and Sannikov (2014)). An extension of our model with financial intermediation would then suggest that banks, just like borrowers in our current model, would have too much leverage and too little insurance in the run-up to a liquidity trap. There would be some room for macroprudential policies that restrict banks' leverage and risks, precisely because these policies would improve aggregate demand and output during the liquidity trap (see, for example, the discussion in Jeanne and Korinek, 2014). We leave a formal analysis of macroprudential regulation of banks in environments with aggregate demand externalities for future work.

A growing literature on financial crises has emphasized various other factors that encourage excessive leverage, including fire-sale externalities, optimism, and moral hazard. Our analysis suggests these distortions are complementary to the aggregate demand externalities that we emphasize. For instance, asset fire sales reduce aggregate demand by tightening borrowing constraints, which in turn exacerbates aggregate demand externalities. Similarly, optimistic beliefs imply households take on excessive leverage and do not want to insure, which makes it more likely that the economy enters the high-leverage conditions under which aggregate demand externalities matter. An interesting future direction is to investigate further the interaction between various sources of excessive leverage.

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A Appendix: Omitted proofs (for online publication)

A.1 Proofs for the baseline model

This section presents the proofs of the results for the baseline model and its variants analyzed in Sections 3, 4, 5, and 6.

Proving the constrained efficiency of the myopic monetary policy. Suppose the monetary authority does not have commitment power and follows the monetary policy in (10) for each date $t+1 \geq 2$. We claim that it is also constrained efficient to follow this policy at time $t \geq 1$, in the sense that the monetary authority cannot improve all households' welfare by deviating from it. This establishes the constrained efficiency of the policy at each $t \geq 1$. Proposition 6 in Section 6 establishes further the constrained efficiency of the policy at date 0 (once macroprudential policies are in place).

To prove the claim, first consider dates $t \geq 2$. Consider r_{t+1} is sufficiently close to r^*_{t+1} so that the borrowers are constrained (the other case is similar). Then, the allocations for date $t+1$ onwards are the same as the frictionless benchmark in (11), which do not depend on r_{t+1}. In addition, the allocations at date t are subject to the feasibility constraints:

$$c^b_t + c^l_t = 2e_t \leq 2e^*. \tag{A.1}$$

Here, the inequality follows since e^* maximizes the net income per household given the technology [cf. Eq. (7)]. Note also that setting $r_{t+1} = r^*_{t+1}$ obtains the upper bound in (A.1). Thus, deviating from this policy cannot improve the welfare of one group of households without hurting the others.

Next consider date $t=1$ for the case $r^*_2 < 0$ (the case $r^*_2 \geq 0$ is identical to the above analysis). In this case, the monetary authority faces the additional constraint, $r_2 \geq 0$. The same steps as above then imply a tighter feasibility constraint:

$$c^b_1 + c^l_1 = 2e_1 \leq e_1 - (d_1 - \phi) + \bar{c}^l_1 < 2e^*.$$

Moreover, setting $r_2 = 0$ obtains the upper bound (in the weak inequality). Thus, the policy in (10) is also constrained efficient at this date, proving the claim. □

Proof of Lemma 1. (i) If $r_{t+1} > 0$, then, by assumption, monetary policy does not face any constraints and is able to implement the efficient level of output $e_t = e^*$. (ii) If $r_{t+1} = 0$, then the frictionless interest rate satisfies $r^*_t \leq 0$, which further implies $e_t = \frac{c^b_t + c^l_t}{2} \leq e^*$. □

Proof of Proposition 1. The result claims there is a recession at date 1 under appropriate conditions. To prove this result, suppose the contrary, that is, $e_0 = e_1 = e^*$. Let $\bar{r}_1(d_0)$ denote the interest rate at which lenders would hold assets $\bar{d}_1$ (more precisely, debt $-\bar{d}_1$) in equilibrium starting with initial assets d_0, defined by:

$$\frac{1}{1+\bar{r}_1} = \frac{\beta^l u'\left(e^* + \bar{d}_1 - \phi\right)}{u'\left(e^* + d_0 - \bar{d}_1/\left(1+\bar{r}_1\right)\right)}. \tag{A.2}$$

Note that $\bar{r}_1(d_0)$ is a decreasing function of d_0. This also implies that $d_0 - \bar{d}_1/(1+\bar{r}_1(d_0))$ is increasing in d_0.

The equilibrium features $d_1 > \bar{d}_1$ if the marginal rates of substitution of the two types of households at the debt level $\bar{d}_1$ satisfy:

$$
\begin{aligned}
& \left.\frac{\beta^l u'\left(c_1^l\right)}{u'\left(c_0^l\right)}\right|_{d_1=\bar{d}_1} > \left.\frac{\beta^b u'\left(c_1^b\right)}{u'\left(c_0^b\right)}\right|_{d_1=\bar{d}_1} \\
\text{or} \quad & \frac{\beta^l u'\left(e^* + \bar{d}_1 - \phi\right)}{u'\left(e^* + d_0 - \bar{d}_1/\left(1+\bar{r}_1\left(d_0\right)\right)\right)} > \frac{\beta^b u'\left(e^* - \bar{d}_1 + \phi\right)}{u'\left(e^* - d_0 + \bar{d}_1/\left(1+\bar{r}_1\left(d_0\right)\right)\right)}. \qquad \text{(A.3)}
\end{aligned}
$$

Observe that the right hand side of this inequality is decreasing in β^b. Hence, for a given debt level d_0, there is a threshold level of impatience $\bar{\beta}^b(d_0)$ such that the inequality holds for each $\beta^b \geq \bar{\beta}^b(d_0)$.

Similarly, since $d_0 - \bar{d}_1/(1+\bar{r}_1(d_0))$ is increasing in d_0, the left hand side of (A.3) is increasing in d_0, while the right hand side is decreasing in d_0. Hence, for a given level β^b, there is a threshold level $\bar{d}_0\left(\beta^b\right)$ such that the inequality holds for each $d_0 > \bar{d}_0\left(\beta^b\right)$. It follows that $d_1 > \bar{d}_1$, and thus, there is a recession at date 1, if the borrowers is sufficiently impatient or sufficiently indebted at date 0. □

Characterizing the upper bound in Assumption (1). The upper bound $\tilde{d}_1$ is the level of debt that triggers a liquidity trap not only at date 1 but also at date 0. To characterize this level, consider the lenders' optimality condition (A.2) corresponding to the interest rate $r_1 = 0$. Rewriting this expression, $\tilde{d}_0$ is found as the unique solution to:

$$
u'\left(e^* + \tilde{d}_0 - \bar{d}_1\right) = \beta^l u'\left(e^* + \bar{d}_1 - \phi\right).
$$

It can be checked that the competitive equilibrium features $r_1 > 0$ if and only if $d_0 < \tilde{d}_0$. It can also be checked that $\tilde{d}_0 > \bar{d}_0\left(\beta^b\right)$, where $\bar{d}_0\left(\beta^b\right)$ is the threshold level of debt that triggers a liquidity trap at date 1, which is characterized in the proof of Proposition **1.** Hence, for each $\beta^b \leq \beta^l$, there is a non-empty set of initial debt levels $d_0 \in \left(\bar{d}_0\left(\beta^b\right), \tilde{d}_0\right]$ that triggers a recession at date 1 but not at date 0. Hence, Proposition 1 applies for a non-empty set of parameters. □

Proof of Lemma 2. First consider the case $d_1 > \bar{d}_1$. Eq. (15) implies $\frac{\mathbf{d}e_1}{\mathbf{d}d_1} = -1$. Eq. (17) then implies $\frac{\mathbf{d}V^h}{\mathbf{d}d_1} = -u'\left(u_1^h\right) < 0$.

Next consider the case $d_1 < \bar{d}_1$. In this case, differentiating lenders' Euler equation (16), we have:

$$
\frac{\mathbf{d}r_2}{\mathbf{d}d_1} = \frac{u''\left(c_1^l\right)}{\beta^l u'\left(c_2^l\right) - u''\left(c_1^l\right)\phi/\left(1+r_2\right)^2} < 0.
$$

The change in borrowers' consumption, $\frac{\mathbf{d}c_1^b}{\mathbf{d}d_1}$, is then given by

$$\eta=\frac{\phi}{(1+r_2)^2}\cdot\frac{-\mathbf{d}r_2}{\mathbf{d}d_1}=\frac{\phi}{(1+r_2)^2}\frac{-u''\left(c_1^l\right)}{\beta^l u'\left(c_2^l\right)-u''\left(c_1^l\right)\phi/\left(1+r_2\right)^2},\tag{A.4}$$

whereas the change in lenders' consumption satisfies $\frac{\mathbf{d}c_1^l}{\mathbf{d}d_1}=-\eta$. It can also be checked that $\eta\in(0,1)$, completing the proof. □

Proof of Proposition 2. Let $\nabla_{sub}V^h\left(d_1^h,d_1\right)$ denote the set of subgradients of function $V^h(\cdot)$ with respect to its second variable (aggregate debt level). If $d_1\neq\bar{d}_1$, the function V^h is differentiable in its second variable. In this case, there is a unique subgradient characterized by Lemma 2. If $d_1=\bar{d}_1$, then the function V^h has a kink at $\bar{d}_1$ due to the kink of the function $e_1(d_1)$ (see Eqs. (17) and (15)). In this case, there are multiple subgradients each for borrowers and lenders, characterized by:

$$\begin{aligned}\nabla_{sub}V^b\left(d_1^b,\bar{d}_1\right)&=\left[-u'\left(c_1^b\right),\quad \eta u'\left(c_1^b\right)\right],\\ \nabla_{sub}V^l\left(d_1^l,\bar{d}_1\right)&=\left[-u'\left(c_1^l\right),-\eta u'\left(c_1^l\right)\right].\end{aligned}\tag{A.5}$$

In particular, for each h, the subgradients lie in the interval between the right and the left derivatives of the function V^h characterized in Lemma 2.

Next consider the optimality conditions for problem (19), which can be written as:

$$\frac{\beta^l\left[u'\left(c_1^l\right)+\delta^l\right]}{u'\left(c_0^l\right)}=\frac{\beta^b\left[u'\left(c_1^b\right)-\delta^b\right]}{u'\left(c_0^b\right)},\tag{A.6}$$

where $\delta^h\in\nabla_{sub}V^h\left(d_1^h,d_1\right)$ denotes the subgradient evaluated at $d_1=d_1^b=-d_1^l$. Note that we consider generalized first order conditions that apply also at points at which the objective function might have a kink. Conversely, it can also be seen that any allocation that satisfies these conditions, along with the intratemporal condition $v'\left(n_0^h\right)=1$ for each h, corresponds to a solution to problem (19) given Pareto weights that satisfy $\frac{\gamma^b}{\gamma^l}=\frac{u'(c_0^b)}{u'(c_0^l)}$. Hence, it suffices to characterize the allocations that satisfy condition (A.6).

First consider the case $d_1<\bar{d}_1$. Using Lemma 2, condition (A.6) becomes identical to the Euler equations (16), proving the first part. Next consider the case $d_1>\bar{d}_1$. Using Lemma 2, condition (A.6) is violated since the left hand side is zero while the right hand side is strictly positive. Hence, there is no constrained efficient allocation with $d_1>\bar{d}_1$. Finally consider the case $d_1=\bar{d}_1$. Using (A.5), we have:

$$\frac{\beta^l\left[u'\left(c_1^l\right)+\delta^l\right]}{u'\left(c_0^l\right)}\in\left[0,\frac{(1-\eta)\,\beta^l u'\left(c_1^l\right)}{u'\left(c_0^l\right)}\right]$$

and:

$$\frac{\beta^b\left[u'\left(c_1^b\right)-\delta^b\right]}{u'\left(c_0^b\right)}\in\left[\frac{(1-\eta)\,\beta^b u'\left(c_1^b\right)}{u'\left(c_0^b\right)},\frac{2\beta^b u'\left(c_1^b\right)}{u'\left(c_0^b\right)}\right].$$

Combining these expressions with condition (A.6), we obtain $\frac{\beta^l u'\left(c_1^l\right)}{u'\left(c_0^l\right)}\geq\frac{\beta^b u'\left(c_1^b\right)}{u'\left(c_0^b\right)}$. Conversely, for any allocation that satisfies this inequality, there exists subgradients δ^l and δ^b such that condition (A.6) holds. It follows that the optimal allocations with $d_1 \geq \bar{d}_1$ are characterized by $d_1 = \bar{d}_1$ and the inequality (20), completing the proof. $\square$

Proof of Corollary 1. Note from the discussion in Section 4 that lowering the debt level to $\bar{d}_1$ generates an ex-post Pareto improvement relative to the competitive equilibrium allocation in Proposition 1. It follows that setting the debt level $d_1 = \bar{d}_1$ while keeping the ex-ante allocations the same, $\left(c_0^h, n_0^h\right)_h = \left(c_0^{h,eq}, n_0^{h,eq}\right)_h$, yields an ex-ante Pareto improvement. Specifically, this allocation strictly improves borrowers' welfare while leaving lenders indifferent. $\square$

Proof of Corollary 2. Consider constrained efficient allocations $\left(\left(\left(c_0^h, n_0^h\right)\right)_h, d_1\right)$ with $d_1 \geq \bar{d}_1$ (the other case is straightforward) and $u'\left(c_0^l\right) \geq \beta^l u'\left(c_1^l\right)$. Note, from our analysis in Section 3, that these allocations feature $e_1 = e^*$ and:

$$c_1^l = \bar{c}_1^l = e^* + \left(\bar{d}_1 - \phi\right) \text{ and } c_1^b = e^* - \left(\bar{d}_1 - \phi\right). \tag{A.7}$$

We thus have a collection $\left(c_0^l, c_0^b, c_1^l, c_1^b\right)$ that satisfies the inequality (20) and Eq. (A.7), along with the resource constraints at date 0 as well as $c_0^l < \bar{c}_0^l$.

We first prove that this allocation can be implemented with the debt limit $d_1^h \leq \bar{d}_1$, and an appropriate transfer T_0^b. The debt limit does not bind for lenders, so that the interest rate is given by $\frac{1}{1+r_1} = \frac{\beta^l u'\left(c_1^l\right)}{u'\left(c_0^l\right)}$. Given this interest rate, let T_0^b denote the unique transfer that ensures lenders' date 0 budget constraint holds as equality, that is, the unique solution to:

$$c_0^l = e^* + \left(d_0 - T_0^b\right) - \frac{\bar{d}_1}{1+r_1}.$$

With this transfer, lenders optimally choose $d_1^l = -\bar{d}_1$. Given the inequality in (20), borrowers are constrained and they also optimally choose $d_1^b = \bar{d}_1$. It follows that the debt limit along with the transfer T_0^b implements the constrained efficient allocation. Note also that the implementation does not violate the lower bound constraint (6) at date 0 because of the assumption $u'\left(c_0^l\right) \geq \beta^l u'\left(c_1^l\right)$.

We next show that the same allocation can also be implemented with the tax policy, τ_0^b. In this case, both borrowers and lenders are unconstrained and their Euler equations imply

$$\frac{1}{1+r_1} = \frac{\beta^l u'\left(c_1^l\right)}{u'\left(c_0^l\right)} = \frac{\beta^b u'\left(c_1^b\right)}{u'\left(c_0^b\right)} \frac{1}{1-\tau_0^b},$$

which also pins down the pre-tax interest rate r_1. Moreover, lenders' budget constraint at date 0 can be written as:

$$c_0^l = e^* + \left(d_0 - T_0^b\right) - \frac{\bar{d}_1}{1+r_1} + \frac{\bar{d}_1}{1+r_1} \frac{\tau_0^b}{2},$$

where the last term captures the lump-sum rebates from the tax policy, and borrowers have a similar budget constraint. Let T_0^b denote the unique solution to lenders' budget constraint. With this transfer, and in view of Eq. (21), households optimally choose $d_1^b = -d_1^l = \bar{d}_1$. Thus, the tax policy, along with an appropriate transfer, also implements the constrained efficient allocation. Note that the implementation does not violate the lower bound constraint because the net-of-tax interest rates for the two groups of households satisfy $\frac{1+r_1}{1-\tau_0^b} \geq 1 + r_1 \geq 1$ in view of the assumption $u'\left(c_0^l\right) \geq \beta^l u'\left(c_1^l\right)$. $\square$

Proof of Proposition 3. Under either condition (i), (ii), or (iii), we claim that there exists an equilibrium in which $d_{1,L} \geq \bar{d}_1$ and a recession is triggered in state L of date 1. By using the resource constraints and substituting $m_L = d_1 - d_{1,L}$, the optimality conditions (for the conjectured equilibrium) can be written as:

$$
\begin{aligned}
q_L &= \frac{\beta^l \pi_L^l u'\left(\bar{c}_1^l\right)}{u'\left(e^* + d_0 - \frac{d_1}{1+r_1} + q_L\left(d_1 - d_{1,L}\right)\right)} = \frac{\beta^b \pi_L^b u'\left(\bar{c}_1^l + 2\left(\phi - d_{1,L}\right)\right)}{u'\left(e^* - \left(d_0 - \frac{d_1}{1+r_1}\right) - q_L\left(d_1 - d_{1,L}\right)\right)} \\
\frac{q_L}{\frac{1}{1+r_1} - q_L} &= \frac{\pi_L^l}{1-\pi_L^l} \frac{u'\left(\bar{c}_1^l\right)}{u'\left(e^* + \left(1-\beta^l\right) d_1\right)} = \frac{\pi_L^b}{1-\pi_L^b} \frac{u'\left(\bar{c}_1^l + 2\left(\phi - d_{1,L}\right)\right)}{u'\left(e^* - \left(1-\beta^l\right) d_1\right)}.
\end{aligned}
$$

Here, the second optimality condition is obtained by combining the two optimality conditions (27) and (28), and illustrates that households equate price to probability ratios between states L and H. The expression $q_H \equiv \frac{1}{1+r_1} - q_L$ is the shadow price for the H-state Arrow-Debreu security.

The previously displayed expressions represent 4 equations in 4 unknowns, $d_1, d_{1,L}, \frac{1}{1+r_1}, q_L$. Given the regularity conditions, there is a unique solution. For the conjectured allocation to be an equilibrium, we also need the solution to satisfy $d_{1,L} \geq \bar{d}_1$. First consider conditions (i) or (ii), i.e., suppose $\pi_L^b = \pi_L^l$. In this case, a similar analysis as in the proof of Proposition 3 establishes that $d_{1,L} \geq \bar{d}_1$ is satisfied when $\beta^b \leq \bar{\beta}^b\left(d_0, \pi_L^b\right)$ or when $d_0 \geq \bar{d}_0\left(\beta^b, \pi_L^b\right)$ (for appropriate threshold functions $\bar{\beta}^b(\cdot)$ and $\bar{d}_0(\cdot)$). Next consider condition (iii). It can be checked that $d_{1,L}$ is decreasing in π_L^b, and that $\lim_{\pi_L^b \to 0} d_{1,L} > \bar{d}_1$ (since $\lim_{\pi_L^b \to 0} c_{1,L}^b = 0$). Thus, there exists a threshold function $\bar{\pi}_L^b\left(\beta^b, d_0\right)$ such that $d_{1,L} > \bar{d}_1$ whenever $\pi_L^b < \bar{\pi}_L^b\left(\beta^b, d_0\right)$, completing the proof. $\square$

Proof of Proposition 4. The proof proceeds similar to the proof of Proposition 2. The optimality conditions for problem (29) imply $\frac{\beta^l \pi_H^l u'\left(c_{1,H}^l\right)}{u'\left(c_0^l\right)} = \frac{\beta^b \pi_H^b u'\left(c_{1,H}^b\right)}{u'\left(c_0^b\right)}$ and

$$
\frac{\beta^l \pi_L^l \left[u'(c_{1,L}^l) + \delta^l\right]}{u'\left(c_0^l\right)} = \frac{\beta^b \pi_L^l \left[u'(c_{1,L}^b) - \delta^b\right]}{u'\left(c_0^b\right)}, \tag{A.8}
$$

Here, δ^l and δ^b denote respectively the subgradients of $V^h\left(d_{1,L}^h, d_{1,L}\right)$ with respect to the second variable, evaluated at $d_{1,L} = d_{1,L}^b = -d_{1,L}^l$. Conversely, it can be seen that any allocation that satisfies these equations corresponds to a solution to the planner's problem with appropriate Pareto weights. Hence, it suffices to characterize the allocations that satisfy condition (A.6).

For the case $d_{1,L} < \bar{d}_1$, applying Lemma 2 in the numerator of Eq. (A.8) makes the condition equivalent to (28). For the case $d_{1,L} > \bar{d}_1$, the numerator on the left hand side of Eq. (A.8) is zero whereas the right-hand side remains positive, implying that $d_{1,L} > \bar{d}_1$ is never optimal. Finally, for the case $d_{1,L} = \bar{d}_1$, condition (A.8) implies the insurance inequality (30). Conversely, given the inequality in (30), there exists a subgradient such that the condition (A.8) holds. This completes the characterization of the solution to problem (29). □

Proof of Corollary 3. Follows from very similar steps as in the proof of Corollary 1. □

Proof of Corollary 4. Follows from very similar steps as in the proof of Corollary 2. □

We next establish the following lemma, which will be useful to prove Proposition 5.

Lemma 3. *Consider a strictly increasing and strictly concave function $u(\cdot)$ such that $-u''(x)/u'(x)$ is a weakly decreasing function of x. Then,*

$$\frac{\mathbf{d}}{\mathbf{d}x}\left(\frac{u'(x+y)}{u'(x-y)}\right) \geq 0 \text{ and } \frac{\mathbf{d}}{\mathbf{d}y}\left(\frac{u'(x+y)}{u'(x-y)}\right) < 0,$$

for each $x, y \in \mathbb{R}_+$.

Proof of Lemma 3. Note that

$$\frac{\mathbf{d}}{\mathbf{d}x}\left(\frac{u'(x+y)}{u'(x-y)}\right) = \frac{u'(x+y)}{u'(x-y)}\left(\frac{u''(x+y)}{u'(x+y)} - \frac{u''(x-y)}{u'(x-y)}\right) \geq 0,$$

where the inequality follows since $-u''(x)/u'(x)$ is weakly decreasing in x. We also have:

$$\frac{\mathbf{d}}{\mathbf{d}y}\left(\frac{u'(x+y)}{u'(x-y)}\right) = \frac{u''(x+y)\,u'(x-y) + u'(x+y)\,u''(x-y)}{u'(x-y)^2} < 0,$$

where the inequality follows since $u(\cdot)$ is strictly concave. □

Proof of Proposition 5. Let $d_1(r_1)$ and $e_0(r_1)$ denote the solution to Eqs. (35). It is also useful to define $y(r_1) = d_0 - \frac{d_1(r_1)}{1+r_1}$, which corresponds to lenders' consumption at date 0 in excess of their net income. Note that, by assumption, we have $y(r_1) > 0$.

We first show that $e_0'(r_1) < 0$. Suppose, to reach a contradiction, $e_0'(r_1) \geq 0$. Note that lenders' Euler equation implies [cf. Eq. (35)]:

$$\frac{1}{1+r_1}u'(e_0(r_1) + y(r_1)) = \beta^l u'(\bar{c}_1^l). \tag{A.9}$$

Since $e_0'(r_1) \geq 0$, this expression implies $y'(r_1) < 0$. Using the definition of $y(\cdot)$, this further implies $d_1'(r_1) > 0$. Next consider borrowers' Euler equation [cf. Eq. (35)]:

$$\frac{1}{1+r_1} = \frac{\beta^b u'\left(\bar{c}_1^l - 2\left(d_1(r_1) - \phi\right)\right)}{u'(e_0(r_1) - y(r_1))}.$$

The left hand side is strictly decreasing in r_1. However, since $e_0'(r_1) \geq 0, y'(r_1) < 0$ and $d_1'(r_1) > 0$, the right hand side is strictly increasing in r_1. This yields a contradiction and proves $e_0'(r_1) < 0$.

We next establish that $d_1'(r_1) > 0$. Suppose, to reach a contradiction, $d_1'(r_1) \leq 0$. Using the definition of $y(\cdot)$, this further implies $y'(r_1) > 0$. Combining lenders' and borrowers' Euler equations, we also have [cf. Eq. (35)]:

$$\frac{\beta^l}{\beta^b} \frac{u'\left(\bar{c}_1^l\right)}{u'\left(\bar{c}_1^l - 2\left(d_1(r_1) - \phi\right)\right)} = \frac{u'\left(e_0(r_1) + y(r_1)\right)}{u'\left(e_0(r_1) - y(r_1)\right)}.$$

The left hand side is weakly increasing in r_1 since $d_1'(r_1) \leq 0$. However, since $e_0'(r_1) < 0$ and $y'(r_1) > 0$, Lemma 3 implies the right hand side is strictly decreasing in r_1. This yields a contradiction and shows $d_1'(r_1) > 0$, completing the proof. □

Proof of Proposition 6. This planner also faces the constrained planning problem (2) after replacing $u\left(c_0^h\right)$ with a more general utility function, $u_0^h\left(c_0^h\right)$. The same steps as in the proof of Proposition 2 show that the constrained efficient allocations always feature the maximum level of net income, $e_0 = e^*$. Implementing this outcome requires setting $r_1 = r_1^*$, proving the result. □

A.2 Proofs for the extension with flexible MPC differences

This section develops the extension with flexible MPC differences between borrowers and lenders, described and used in Sections 4.3,5.1, and 5.2. We first consider the case without uncertainty, completing the characterization in Section 4.3. We then introduce uncertainty, with and without complete markets, and complete the characterizations in respectively Sections 5.1 and 5.2.

A.2.1 Case without uncertainty

We analyze a slightly more general version of the model than described in Section 4.3. In particular, suppose there are several groups of households denoted by $h = \{1, .., |H|\}$ each of which has mass ω^h and log-utility. Each individual household experiences a shock at date 1 which turns her into one of two types, $\{h_{unc}, h_{con}\}$. As in the main text, type h_{unc} households are unconstrained and thus they have a low MPC equal to $1 - \beta$. Type h_{con} households mechanically target a constrained debt level of $d_2^{h_{con}} = \phi^h$. Consequently, these households have a high MPC equal to 1. Groups differ in terms of the fraction of the high MPC types they contain, $\alpha^h \in [0, 1]$, as well as the initial debt level, d_1^h. As before, the type shocks are uninsurable at date 0, so that households within the same group have the same level of debt at date 1, $d_1^{h_{unc}} = d_1^{h_{con}} = d_1^h$.

We assume that group 1 does not feature any constrained households $\alpha^1 = 0$, and brings assets to date 1, $d_1^1 < 0$. This group captures fully unconstrained lenders as in the main text. The remaining groups can capture heterogeneous groups of borrowers with different MPCs, generalizing the borrowers in the main text. These groups can also capture lenders that might have higher MPCs at date 1 than the baseline lenders (perhaps because they have relatively low assets and can become constrained with some probability). The model in Section 4.3 is a special case with two groups of households: unconstrained lenders with mass $\omega^1 = \omega$ and a single group of borrowers with mass $\omega^2 = 1$ and constraints $\alpha^2 = \alpha$.

We next analyze the general model and obtain the results in Section 4.3 as a special case. As before, under appropriate conditions (that will be characterized below), date 1 features a liquidity trap equilibrium with $r_2 = 0$ and $e_1 \leq e^*$. In this equilibrium, type h_{con} households are forced into deleveraging. Thus, their outstanding debt for the next date is

$$d_2^{h_{con}} = \phi^h. \tag{A.10}$$

In contrast, type h_{unc} households choose their consumption and outstanding debt according to the Euler equation,

$$u'\left(c_1^{h_{unc}}\right) = \beta u'\left(c_2^{h_{unc}}\right), \text{ with } c_1^{h_{unc}} = e_1 - d_1^h + d_2^{h_{unc}} \text{ and } c_2^{h_{unc}} = e^* - d_2^{h_{unc}}\left(1-\beta\right).$$

After substituting log-utility and rearranging terms, these households' debt choices satisfy

$$e^* - d_2^{h_{unc}} = \beta\left(e_1 - d_1^h\right). \tag{A.11}$$

Finally, the relevant debt market clearing conditions can be written as:

$$\begin{aligned} \sum_h \omega^h \left[\alpha^h d_1^h + \left(1-\alpha^h\right) d_1^h\right] &= 0, && \text{(A.12)} \\ \text{and } \sum_h \omega^h \left[\alpha^h d_2^{h_{con}} + \left(1-\alpha^h\right) d_2^{h_{unc}}\right] &= 0. && \text{(A.13)} \end{aligned}$$

The equilibrium is solved by combining Eqs. (A.10)-(A.13), which gives:

$$\left(e^* - \beta e_1\right) \sum_h \left(1-\alpha^h\right)\omega^h = \sum_h \alpha^h \omega^h \left(\beta d_1^h - \phi^h\right). \tag{A.14}$$

Roughly speaking, the right hand side of this expression provides a measure of aggregate deleveraging, that is, the reduction in debt level by all constrained (or high MPC) households. The left hand side captures the adjustment by all unconstrained (or low MPC) households. The economy experiences a liquidity trap as long as the right hand side is sufficiently large, that is,

$$\sum_h \alpha^h \omega^h \left(\beta d_1^h - \phi^h\right) > e^*\left(1-\beta\right) \sum_h \left(1-\alpha^h\right)\omega^h. \tag{A.15}$$

Note that the liquidity trap is triggered when the constrained groups' initial debt level is large. When this is the case, Eq. (A.14) pins down the equilibrium level of net income as a function of each group's initial debt level:

$$e_1\left(\left\{d_1^h\right\}_h\right) = \frac{e^*}{\beta} - \sum_h \frac{\omega^h \alpha^h}{\sum_h \left(1-\alpha^h\right)\omega^h}\left(d_1^h - \frac{\phi^h}{\beta}\right). \tag{A.16}$$

Let $\bar{\omega} = \sum_h \omega^h$ denote the total mass of households and $\bar{\alpha} = \sum_h \frac{\omega^h}{\bar{\omega}} \alpha^h$ denote the average fraction of high MPC households. Then, using Eq. (A.16), we obtain, $\frac{\bar{\omega}}{\omega^h} \frac{\mathbf{d}e_1}{\mathbf{d}d_1^h} = -\frac{\alpha^h}{1-\bar{\alpha}}$. To convert this expression into MPCs, note that $\alpha^h = \frac{MPC^h - MPC^1}{1-MPC^1}$, where we recall that group 1 captures fully unconstrained lenders. Plugging in, we obtain:

$$\frac{\bar{\omega}}{\omega^h} \frac{\mathbf{d}e_1}{\mathbf{d}d_1^h} = -\frac{\alpha^h}{1-\bar{\alpha}} = \frac{MPC^h - MPC^1}{1 - \overline{MPC}}. \tag{A.17}$$

The left hand side captures the effect of raising group h households' total debt by 1 unit on total net demand. The expression is normalized by $\bar{\omega}/\omega^h$ since the group's total debt level is $\omega^h d_1^h$ and the total net demand (or income) is $\bar{\omega} e_1$. Intuitively, increasing the debt of group h effectively transfers financial wealth from this group to unconstrained households. This in turn lowers aggregate demand, and more so when group h has high MPC relative to unconstrained households. The effect is amplified by the Keynesian income multiplier, captured by the denominator of the right-hand side.

To obtain Eq. (A.16), note that in the special case of Section 4.3 the equilibrium debt level satisfies $d_1 = d_1^2 = -d_1^1/\omega$. Thus, raising *the equilibrium* debt level amounts to raising the debt level of group 2 (borrowers) while reducing the debt level of group 1 (lenders). In particular, we have:

$$\bar{\omega} \frac{\mathbf{d}e_1}{\mathbf{d}d_1} = \bar{\omega} \left(\frac{\mathbf{d}e_1}{\mathbf{d}d_1^2} - \omega \frac{\mathbf{d}e_1}{\mathbf{d}d_1^1} \right) = \bar{\omega} \frac{\mathbf{d}e_1}{\mathbf{d}d_1^2} = \frac{MPC^2 - MPC^1}{1 - \overline{MPC}}.$$

This gives the expression in (A.16) after relabeling $b = 2$ and $l = 1$. Notice that the effect through lenders' debt (or financial wealth) drops out, because those households are unconstrained by assumption, which implies $\frac{\mathbf{d}e_1}{\mathbf{d}d_1^1} = 0$ [cf. Eq. (A.17)].

We next characterize the constrained optimal allocations for the range in which there is a liquidity trap. We use $V^h \left(d_1^h, e_1\right)$ to denote their continuation utility conditional on their debt level d_1^h and the aggregate net income e_1. With some abuse of notation, we also let $u'\left(c_1^h\right) = \alpha^h u'\left(c_1^{h_{con}}\right) + \left(1-\alpha^h\right) u'\left(c_1^{h_{unc}}\right)$ denote group h households' expected marginal utility at date 1 before the realization of their types. The constrained planning problem can be written as:

$$\max_{\left(c_0^h, d_1^h\right)_h} \sum_h \gamma^h \omega^h \left[u\left(c_0^h\right) + \beta^h V^h \left(d_1^h, e_1\right) \right] \tag{A.18}$$

$$\text{such that} \quad \sum_h \omega^h d_1^h = 0 \text{ for each } h, \text{ and } \sum_h \omega^h c_0^h = \sum_h \omega^h e^*.$$

Let A and B respectively denote the Lagrange multipliers for the constraints. The first order conditions can then be written as $\gamma^h u'\left(c_0^h\right) = B$ for each h, and

$$-\gamma^h \omega^h \beta^h u'\left(c_1^h\right) + \left[\sum_{\tilde{h}} \gamma^{\tilde{h}} \omega^{\tilde{h}} \beta^{\tilde{h}} u'\left(c_1^{\tilde{h}}\right) \right] \frac{\mathbf{d}e_1}{\mathbf{d}d_1^h} = A\omega^h.$$

Combining the first order conditions, and using the fact that $\frac{de_1}{dd_1^1} = 0$ (since group 1 is unconstrained), we obtain

$$\frac{\beta^1 u'\left(c_1^1\right)}{u'\left(c_0^1\right)} = \frac{\beta^h u'\left(c_1^h\right)}{u'\left(c_0^h\right)} - \left[\sum_{\tilde{h}} \frac{\omega^{\tilde{h}}}{\bar{\omega}} \frac{\beta^{\tilde{h}} u'\left(c_1^{\tilde{h}}\right)}{u'\left(c_0^{\tilde{h}}\right)}\right] \left(\frac{\bar{\omega}}{\omega^h} \frac{\mathbf{d}e_1}{\mathbf{d}d_1^h}\right), \tag{A.19}$$

for each h. In particular, the planner penalizes the increase in group h household's total debt to the extent to which this will reduce total net demand, and therefore, the utility of the average household. Note that Eq. (25) in the main text follows a special case.

We finally provide a first order approximation to households' optimal tax rates. Similar to Corollary 2, the optimal allocation can be implemented with linear taxes $\left\{\tau_0^h\right\}_h$ that satisfy $\frac{\beta^1 u'\left(c_1^1\right)}{u'\left(c_0^1\right)} = \frac{\beta^h u'\left(c_1^h\right)}{u'\left(c_0^h\right)} \frac{1}{1-\tau_0^h}$ for each h, with the convention that $\tau_0^1 = 0$. Plugging this into Eq. (A.19), the optimal tax rates solve the system

$$\frac{1}{1-\tau_0^h} = 1 - \left(\frac{\bar{\omega}}{\omega^h} \frac{\mathbf{d}e_1}{\mathbf{d}d_1^h}\right) \sum_{\tilde{h}} \frac{\omega^{\tilde{h}}}{\bar{\omega}} \frac{1}{1-\tau_0^{\tilde{h}}}. \tag{A.20}$$

Taking a first order approximation around zero taxes, we obtain:

$$\tau_0^h \simeq -\frac{\omega^h}{\bar{\omega}} \frac{\mathbf{d}e_1}{\mathbf{d}d_1^h} = \frac{MPC^h - MPC^1}{1 - \overline{MPC}} \text{ for each } h \in \{2, \ldots |H|\}. \tag{A.21}$$

Thus, the tax rate on each group (of borrowers or lenders) is approximately equal to marginal effect of this group's debt on total net demand. This in turn depends on the MPC differences as well as the Keynesian income multiplier. Eq. (26) in the main text follows as a special case.

Note also that the implementation of the optimal policy with the tax rates in (A.21) features two differences from the baseline implementation in Corollary 2. First, the planner uses non-anonymous policies in the sense that the tax rate τ_0^h is only applied to group h households (since different groups require different tax rates). Second, the rate τ_0^h is applied to all debt issuance by this group, d_0^h, as opposed to only positive debt issuance. A tax rate on a negative debt issuance $d_0^h < 0$ is in effect a subsidy for saving: it raises the interest rate households receive to $\frac{1+r_1}{1-\tau_0^h}$. The planner can use these types of subsidies to raise the saving of lenders that have relatively high MPCs at date 1.

A.2.2 Case with uncertainty and complete markets

Next suppose that the economy at date 1 is in two possible states $s \in \{H, L\}$, as described in Section 5. Let $d_{1,H}^h, d_{1,L}^h$ denote group h households' debt level in state H and L, respectively. State H features the frictionless level of net income, $e_{1,H} = e^*$, along with consumption levels, $c_{1,H}^h = e^* + d_{1,H}^h (1-\beta)$ for each h. State L is exactly the same as date 1 of the earlier model so that much of the analysis for the previous case continues to apply. Specifically, the economy experiences a demand-driven recession if the outstanding debt levels $\left\{d_{1,L}^h\right\}_h$

satisfy the inequality in (A.15), and the net income $e_{1,L}$ is characterized by Eq. (A.16).

The main difference in this case concerns the constrained efficient allocations and the optimal macroprudential policies at date 0 (for the range in which there is a liquidity trap in state L). If the market is complete in the sense that households can trade insurance contracts for state L, then the constrained efficient allocations solve the following analogue of problem (A.18):

$$\max_{\left(c_0^h, d_{1,L}^h, d_{1,H}^h\right)_h} \sum_h \gamma^h \omega^h \left[u\left(c_0^h\right) + \beta^h \sum_{s \in \{H,L\}} \pi_s^h V_s^h \left(d_{1,s}^h, e_1\right) \right] \tag{A.22}$$

$$\text{such that} \quad \sum_h \omega^h d_s^h = 0 \text{ for each } h \text{ and } s \in \{H, L\}, \text{ and } \sum_h \omega^h c_0^h = \sum_h \omega^h e^*.$$

The optimality conditions for c_0^h and $\left(d_{1,H}^h\right)_h$ imply full insurance for state H (except for the idiosyncratic shocks of households), that is, $\frac{\beta^1 \pi_H^1 u'(c_{1,H}^1)}{u'(c_0^1)} = \frac{\beta^h \pi_H^h u'(c_{1,H}^h)}{u'(c_0^h)}$ for each h. In contrast, the optimality conditions for c_0^h and $\left(d_{1,L}^h\right)_h$ imply the following analogue of Eq. (A.19):

$$\frac{\beta^1 \pi_L^1 u'\left(c_{1,L}^1\right)}{u'\left(c_0^1\right)} = \frac{\beta^h \pi_L^h u'\left(c_{1,L}^h\right)}{u'\left(c_0^h\right)} - \left[\sum_{\tilde{h}} \frac{\omega^{\tilde{h}}}{\bar{\omega}} \frac{\beta^{\tilde{h}} \pi_L^{\tilde{h}} u'\left(c_{1,L}^{\tilde{h}}\right)}{u'\left(c_0^{\tilde{h}}\right)}\right] \left(\frac{\bar{\omega}}{\omega^h} \frac{\mathbf{d} e_{1,L}}{\mathbf{d} d_{1,L}^h}\right)$$

for each h. The insurance subsidy ζ_0^h [defined as the wedge in Eq. (31)] then solves exactly the same system as the optimal tax in (A.20). Thus, the optimal subsidy on group h satisfies

$$\zeta_0^h \simeq -\frac{\omega^h}{\bar{\omega}} \frac{\mathbf{d} e_{1,L}}{\mathbf{d} d_{1,L}^h} = \frac{MPC^h - MPC^1}{1 - \overline{MPC}} \text{ for each } h \in \{2, .. |H|\}. \tag{A.23}$$

This also implies Eq. (32) in the main text and completes the characterization with complete markets.

A.2.3 Case with uncertainty and incomplete markets

Next consider the case in which the market is incomplete in the sense that state L is uninsurable. In this case, households are constrained to choose $m_L^h = 0$ (or equivalently, $d_{1,L}^h = d_{1,H}^h$). The constrained efficient allocations solve problem (A.22) with the additional restriction that $d_1^h \equiv d_{1,H}^h = d_{1,L}^h$ for each h. The first order conditions imply the following analogue of Eq. (A.19):

$$\frac{\beta^1 E^1 \left[u'\left(c_1^1\right)\right]}{u'\left(c_0^1\right)} = \frac{\beta^h E^h \left[u'\left(c_{1,H}^h\right)\right]}{u'\left(c_0^h\right)} - \left(\frac{\bar{\omega}}{\omega^h} \frac{\mathbf{d} e_{1,L}}{\mathbf{d} d_1^h}\right) \sum_{\tilde{h}} \pi_L^{\tilde{h}} \frac{\omega^{\tilde{h}}}{\bar{\omega}} \frac{\beta^{\tilde{h}} u'\left(c_{1,L}^{\tilde{h}}\right)}{u'\left(c_0^{\tilde{h}}\right)},$$

for each h. This also implies Eq. (33) in the main text.

In this case, we cannot approximate the optimal tax rate due to the market incompleteness. For a back-of-the-envelope calculation, consider the simplifications in the main text, $\pi \equiv \pi_L^h$ and $u'\left(c_{1,L}^h\right) \simeq u'\left(c_{1,L}^h\right)$ for each h. The latter assumption also implies $\frac{\beta^h u'\left(c_{1,L}^h\right)}{u'\left(c_0^h\right)} \simeq \frac{\beta^h E^h\left[u'\left(c_{1,H}^h\right)\right]}{u'\left(c_0^h\right)}$ for each h at the no-tax allocation. Then, the optimal tax satisfies the system $\frac{1}{1-\tau_0^h} = 1 - \left(\frac{\bar{\omega}}{\omega^h}\frac{\mathbf{d}e_{1,L}}{\mathbf{d}d_1^h}\right)\sum_j \frac{\omega^j}{\bar{\omega}}\frac{1}{1-\tau_0^j}$. Linearizing this expression around zero taxes implies

$$\tau_0^h \simeq -\pi \frac{\bar{\omega}}{\omega^h}\frac{\mathbf{d}e_{1,L}}{\mathbf{d}d_1^h} = \pi \cdot \frac{MPC^h - MPC^1}{1 - \overline{MPC}} \text{ for each } h \in \{2, .. |H|\}.$$

Labeling 1 as l and setting $h = b$, this implies Eq. (34) in the main text, completing the analysis with flexible MPCs.

A.3 Proofs for the extension with fire sales

This section completes the characterization of the model with fire sales described in Section 7. To characterize the condition $p_2 \cdot \partial MRS/\partial p_1 < 1$ in more detail, note that

$$\frac{\partial MRS}{\partial p_1} = \frac{-\left(1-\beta\right)\phi u''\left(c_2^b\right)\left[\left(1-\phi\right)u'\left(c_1^b\right) - 2\phi\beta u'\left(c_2^b\right)\right] - \left(1-\phi\right)\phi u''\left(c_1^b\right)u'\left(c_2^b\right)}{\left[\left(1-\phi\right)u'\left(c_1^b\right) + \phi\beta u'\left(c_2^b\right)\right]^2}$$

If we approximate $\beta \approx 1$, all but the last term disappear from the numerator. Furthermore, we approximate $u'\left(c_1^b\right) \approx \beta u'\left(c_2^b\right)$ which holds exactly in the neighborhood of where the constraint becomes binding. This simplifies the expression in the denominator. Taken together,

$$\frac{\partial MRS}{\partial p_1} \approx -\frac{\phi\left(1-\phi\right)u''\left(c_1^b\right)}{u'\left(c_1^b\right)} = \frac{\phi\left(1-\phi\right)}{\sigma c_1^b} < \frac{1}{p_2} \quad \text{or} \quad \phi\left(1-\phi\right) < \sigma\frac{c_1^b}{p_2}$$

where σ is the intertemporal elasticity of substitution. In short, the solution to equation (38) is unique and well-defined if the leverage parameter is sufficiently small compared to the consumption/asset price ratio. If the condition was violated, an infinitesimal increase at date 1 consumption would lead to a discrete upward jump in the asset price and relax the constraint by more than necessary to finance the marginal increase in consumption. This would violate the assumption that the equilibrium exhibits a binding borrowing constraint. Observe also that this is a common type of condition in models of financial amplification to guarantee uniqueness (see e.g. Lorenzoni, 2008; Jeanne and Korinek, 2010b).

B Appendix: Omitted extensions (for online publication)

B.1 Partially sticky prices

Our baseline model features an extreme form of nominal price stickiness. We next develop a version of the model in which prices are partially flexible. We show that, as long as the monetary policy follows an inflation targeting rule, this model yields the same real allocations as the baseline model, up to a first-order approximation. We also allow the inflation target to be greater than zero and show that, while a higher inflation target reduces the incidence of liquidity traps, it does not change our qualitative results.

For simplicity, we work with a stylized version of the New Keynesian model. We denote the gross nominal inflation rate at time t with $\Pi_t = \frac{P_t}{P_{t-1}}$. A fraction s of firms have sticky prices in the sense that they do not reoptimize their price level every period. However, these firms passively index their price changes to the long run inflation rate in the economy, specifically, they set

$$P_t^{sticky} = P_{t-1}\Pi \text{ for each } t,$$

where $\Pi \geq 1$ denotes the (gross) inflation target that will be described below and P_{-1} is given. Given their predetermined price level, these firms solve problem (9) as in the baseline model. The remaining fraction $1-s$ of firms have fully flexible prices, in the sense that they reoptimize their price level every period. These firms solve problem (8), which yields:

$$P_t^{flex} = \frac{\varepsilon}{\varepsilon - 1} P_t w_t \left(1 - \tau\left(n_t\right)\right) = P_t w_t, \tag{B.1}$$

where we have used the assumption that $\tau(n_t) = 1/\varepsilon$ over the relevant range. The baseline model in Section 2 can be thought of as the special case with $s = 1$ and $\Pi = 1$.

Given the Dixit-Stiglitz technology in (4), the nominal price for the final consumption good satisfies $P_t^{1-\varepsilon} = s\left(P_t^{sticky}\right)^{1-\varepsilon} + (1-s)\left(P_t^{flex}\right)^{1-\varepsilon}$. Log-linearizing this equation around an equilibrium in which all firms set the same price level $P_t^{sticky} \simeq P_t^{flex}$, and combining with the earlier pricing equations, we obtain:

$$\log \Pi_t \simeq \frac{1-s}{s} \log w_t + \log \Pi.$$

Log-linearizing the wage level around the frictionless benchmark, we further obtain a variant of the New Keynesian Phillips Curve:[23]

$$\log \Pi_t \simeq \frac{1-s}{s} \frac{v''\left(n^*\right)}{v'\left(n^*\right) n^*} \log\left(\frac{y_t}{y^*}\right) + \log \Pi. \tag{B.2}$$

[23] Specifically, we first log-linearize Eqs. (4) and (5) around the frictionless benchmark (that features $y_t(\nu) = y^*$), which gives $\log(n_t/n^*) \simeq \log(y_t/y^*)$. We then log-linearize the optimal labor supply condition $w_t = v'(n_t)$, which gives $\log w_t \simeq \frac{v''(n^*)}{v'(n^*)n^*} \log(n_t/n^*)$. Plugging these expressions into the inflation equation yields Eq. (B.2).

Note that inflation depends on the current output gap $\log(y_t/y^*)$, in view of the flexible price firms, as well as the long run inflation target, in view of the sticky price firms with inflation indexation. The extent to which the current output gap influences inflation depends on the fraction of flexible firms, $1-s$, as well as the elasticity of marginal costs (or wages) with respect to the changes in employment.

Eq. (B.2) summarizes the behavior of the supply side of the model up to log-linearization. On the demand side, we replace (10) with an inflation targeting monetary policy, specifically, we assume the monetary authority follows the rule:

$$1+i_{t+1}=\begin{cases}\max\left(0,\left(1+r^*_{t+1}\right)\Pi\left(\frac{\Pi_t}{\Pi}\right)^{\psi}\right) & \text{if } t\geq 1,\\ \max\left(0,\left(1+r^*_{t+1}\right)\Pi_1\left(\frac{\Pi_0}{\Pi}\right)^{\psi}\right) & \text{if } t=0.\end{cases} \tag{B.3}$$

Here, r^*_{t+1} is the frictionless real interest rate defined as before, $\Pi\geq 0$ is the gross inflation target, and $\psi>1$ is a coefficient that captures the responsiveness of monetary policy to inflation. This policy attempts to set the real interest rate equal to its frictionless level, r^*_{t+1}, while also keeping inflation at its target level.[24]

We next turn to the characterization of equilibrium. First consider dates $t\geq 2$ at which $r^*_{t+1}=1/\beta^l-1>0$. At these dates, the equilibrium features the frictionless outcomes, $r_{t+1}=r^*_{t+1}$, $y_t=y^*$, $e_t=e^*$, along with inflation equal to its target level, $\Pi_t=\Pi$. This level of inflation corresponds to an equilibrium in view of Eq. (B.3) and the zero output gap, $\log(y_t/y^*)=0$. Intuitively, since the zero lower bound does not bind for any $\tilde{t}\geq 2$, the monetary policy in (B.3) implements the frictionless outcomes while also stabilizing inflation.[25]

Next consider date $t=1$. The key observations is that, since the monetary policy will stabilize inflation starting date 2, households (rationally) expect the inflation to be equal to its target level, that is, $\Pi_2=P_2/P_1=\Pi$. Combining this with the nominal interest rate bound in (3), we obtain a bound on the real rate as in the baseline model:

$$1+r_2=\frac{1+i_2}{\Pi}\geq\frac{1}{\Pi}. \tag{B.4}$$

Note that, when the gross inflation target is equal to 1, the bound is the same as in the main text [cf. (6)]. Hence, in this case the equilibrium allocations are also exactly the same. For the more general case with $\Pi\geq 1$, the bound on the real interest rate is smaller than in the main text [cf. (B.4)]. Thus, a higher inflation target Π reduces the incidence of a liquidity trap. However, the qualitative analysis is the same as in Section 3. Specifically, there is a threshold debt level $\bar{d}_1(\Pi)$, increasing in Π, such that the economy enters a liquidity trap as long as $d_1>\bar{d}_1(\Pi)$.

One difference from the baseline model concerns the behavior of inflation at date 1. In particular, when $d_1>\bar{d}_1(\Pi)$, the economy features a negative output gap, $\log(y_t/y^*)<0$.

[24] We make a distinction for date 0 because, as we will see, this will be the only date at which the expected inflation will deviate from the target (i.e., $\Pi_1\neq\Pi$). The monetary policy at date 0 is adjusted to take this anticipated deviation into account.

[25] We abstract away from the equilibria with self-fulfilling deflationary traps and inflationary panics (see Cochrane, 2011).

Consequently, Eq. (B.2) implies that the realized inflation at date 1 is below the target level, $\Pi_1 < \Pi$. Thus, the liquidity trap episode is also associated with some disinflation.

Finally, consider the equilibrium at date $t = 0$. The real interest rate at this date is also bounded by $1 + r_1 = \frac{1+i_1}{\Pi_1} \geq 1$. We modify the bound $\tilde{d}_0$ in Assumption (1) appropriately so that this constraint does not bind in equilibrium. Under this assumption, the monetary policy in (B.3) implements the frictionless outcome along with $\Pi_0 = P_0/P_{-1} = \Pi$. The equilibrium debt level is characterized by Eq. (16) as before, and features $d_1 > \bar{d}_1(\Pi)$ under conditions analogous to those in Proposition 1.

In sum, if monetary policy follows the rule in (B.3) with $\Pi = 1$, then the model with partially sticky prices yields the same allocations as in the baseline model, up to a first-order approximation. Intuitively, although prices are somewhat flexible at the micro level, the aggregate prices between dates 1 and 2 continue to be sticky in view of the inflation targeting monetary policy. A higher inflation target Π alleviates the liquidity trap by relaxing the bound in (B.4), but it does not change our qualitative results.

By the same logic, alternative monetary policies that increase inflation at date 2 above the target level Π can also alleviate the liquidity trap. Note, however, that these policies are not time consistent in our environment. Specifically, these policies create a dispersion of relative prices between sticky and flexible price firms, which lowers social welfare. If inflation is costly for this (or other) reasons, then a monetary authority without commitment power will find it optimal to follow the policy in (B.3) together with $\Pi = 1$, which will generate the same real allocations as in the baseline model.

In our model, households at date 0 anticipate the path of future inflation and adjust the interest rate charged accordingly. However, if the deleveraging episode is unanticipated, or if it is stochastic and the financial market is incomplete (as in Section 5.2), then there is an additional force, debt deflation, that would aggravate the recession. When households have noncontingent nominal debt and inflation is lower than expected, the real burden of debt is inflated by the falling price level (see Eggertsson and Krugman, 2012, for a formalization in the context of a liquidity trap). This has the power to significantly exacerbate the resulting recession and compound the aggregate demand externalities. As we discuss in more detail in Remark 2 of the main text, the US economy was fortunately spared any significant disinflation during the most recent macroeconomic slump so that the forces of debt deflation were weak.

B.2 Downward wage rigidities

Our baseline model features nominal price stickiness. We next illustrate that assuming downward wage rigidity (as in Eggertsson and Mehrotra (2014) or Schmitt-Grohe and Uribe (2012c))—as opposed to price rigidity—does not change our results, as long as monetary policy follows an inflation targeting rule.

Suppose each final good firm reoptimizes its price every period. Eq. (B.1) then implies that the aggregate price level is given by

$$P_t = W_t \text{ for each } t, \tag{B.5}$$

where W_t denotes the nominal wage level (equivalently, the relative wage level is $w_t = 1$).

Unlike in the earlier analysis, however, the nominal wage level is assumed to be sticky in the downward direction. In particular, wages cannot fall below a norm $\overline{W}_t$. Following Eggertsson and Mehrotra (2014), we also assume $\overline{W}_t = \gamma W_{t-1} + (1-\gamma) P_t v'(n_t)$, where γ is a parameter that captures the degree of price rigidity and $P_{-1} = W_{-1}$ is given. If $\gamma = 1$, then the nominal wage level cannot fall below its level in the last period. For lower levels of γ, the wage level can fall to some extent (when the marginal cost of labor is low) but it is nonetheless constrained.

The key difference of this model concerns the behavior of households' labor supply. If the (nominal) marginal cost of labor is above the norm, $P_t v'(n_t) \geq \overline{W}_t$, then the constraint does not bind. In this case, the equilibrium wage level satisfies $W_t = P_t v'(n_t)$ and the labor supply is competitive. If instead the marginal cost is below the norm, $P_t v'(n_t) < \overline{W}_t$, then the wage level satisfies $W_t = \overline{W}_t$ and labor supply is rationed (symmetrically across all households). Combining the two cases, labor supply can be summarized as:

$$W_t = \max\left\{\overline{W}_t, P_t v'(n_t)\right\}, \text{ where } \overline{W}_t = \gamma W_{t-1} + (1-\gamma) P_t v'(n_t). \tag{B.6}$$

We also assume that monetary policy follows the inflation targeting rule in (B.3). The rest of the equilibrium is unchanged.

The characterization of equilibrium closely parallels the analysis in Section B.1. Specifically, starting date 2 onwards, the inflation targeting monetary policy implements the frictionless outcomes along with the target inflation level, $\Pi_t = \Pi$. Note that the labor market is in equilibrium because the nonnegative inflation target implies $P_t = W_t \geq P_{t-1} = W_{t-1}$ for each t, so that the wage constraint does not bind and $n_t = n^*$ (cf. Eqs. (B.5) and (B.6)).

At date 1, the expected inflation is equal to target as before, $P_2/P_1 = \Pi$, which leads to the bound on the real rate in (B.4). Given this bound, when the debt level exceeds a threshold $d_1 > \bar{d}_1(\Pi)$, the economy cannot replicate the frictionless outcome. In this case, the economy experiences a demand-driven recession as before. The recession puts downward pressure on the wage level (in view of low marginal costs), but the wages can only fall so much. Specifically, the equilibrium at date 1 features $P_1 = W_1 = \overline{W}_1$, which further implies [cf. Eqs. (B.5) and (B.6)]:

$$P_1 = W_1 = \overline{W}_1 = \frac{\gamma P_0}{1 - (1-\gamma) v'(n_1)}. \tag{B.7}$$

In this case, the wage constraint in (B.6) binds and labor supply is rationed. The equilibrium level of employment satisfies $n_1 = y_1 < n^* = y^*$, and it is determined by aggregate demand at the constrained interest rate in (B.4). Thus, the real equilibrium allocations at date 1 are as in Section B.1. Intuitively, the difference in this case is that the shortage of demand is countered by rationing in the labor market (as opposed to rationing in the goods market that translates into low wages and employment). The analysis of the equilibrium at date 0 also closely parallels that in Section B.1, illustrating that our results are robust to allowing for downward wage rigidity.

Note also that Eq. (B.7) implies that the price level at date 1 satisfies, $P_1 < P_0$, because $v'(n_1) < v'(n^*) = 1$. Hence, as in Section B.1, the economy features disinflation (in fact, deflation) at date 1. However, the magnitude of the disinflation at date 1 is different, as it is governed by the degree of rigidities in the labor market as opposed to the goods market.

B.3 Role of monopoly subsidies

Our baseline model features linear subsidies for monopolists that depend on aggregate employment as follows,

$$\tau(n_t) = \begin{cases} 1/\varepsilon, & \text{if } n_t \leq n^* \\ 0, & \text{if } n_t > n^* \end{cases}. \tag{B.8}$$

We first explain why we take away these subsidies for the range $n_t > n^*$. We then generalize our analysis to the case without monopoly subsidies $\tau = T_t = 0$.

To see why we assume (B.8), consider the alternative assumption that $\tau = 1/\varepsilon$ regardless of the employment level. In this case, Proposition 1 features not one but two equilibria (for some parameters). Specifically, when $d_1 > \bar{d}_1$, there is a second equilibrium that has the same net allocations, e_1, c_1^l, c_1^b, but that differs in actual output and employment. The multiplicity emerges because, in view of the GHH preferences, the net income $e_1 = n_1 - v(n_1) < e^*$ can be obtained by two different levels of employment: one that features a recession $n_1^r < n^*$, and another one that features an excessive boom $n_1^b > n^*$. The two equilibria exhibit the same utility $u\left(c_t^h\right)$ for all households and time periods and are thus identical in terms of their welfare implications. However, the boom equilibrium is fragile in the sense that it is an artefact of positive monopoly subsidies. More specifically, once we take away the subsidies for $n_1 > n^*$ as in (B.8), then the boom equilibrium disappears because monopolists' marginal cost exceeds their marginal product, that is, $w_1 = v'\left(n_1^b\right) > v'(n^*) = 1$. We assume $(B.8)$ since this enables us to focus on the recession equilibrium, which is less fragile (in the sense that it does not depend on the subsidies), while also providing a clean conceptual benchmark for welfare analysis.

We next consider the case without monopoly subsidies $\tau = T_t = 0$ and show that our analysis remains unchanged up to some relabeling—except for the fact that there is now a unique equilibrium with recession at date 1. The main difference in this case concerns the frictionless employment and net income levels, which we respectively denote by n^{**} and e^{**} to emphasize their difference from the efficient levels n^* and e^*. Specifically, the optimality condition for problem (8) implies $1 = p_t = \frac{\varepsilon}{\varepsilon-1} w_t = \frac{\varepsilon}{\varepsilon-1} v'(n^{**})$. This in turn implies $n^{**} < n^*$ and $e^* < e^{**}$ [cf. (7)]. We assume that the monetary policy continues to follow (10) after replacing e^* with e^{**}.[26] With this assumption, the analysis in Section 3, including Proposition 1, remains unchanged after replacing e^* with e^{**}.

Moreover, the equilibrium at date 1 is actually unique in this case, not only in terms of net income but also in terms of actual output and employment. To see this, note that the equilibrium employment with $\tau = 0$ must satisfy $n_1 \leq n^*$, since otherwise monopolists would make negative profits [cf. problem (9)]. Next note that, given any feasible level of net income $e_1 = n_1 - v(n_1)$, there is a unique level of employment that also satisfies $n_1 \leq n^*$. When the debt level is below the threshold $d_1 \leq \bar{d}_1$, the unique equilibrium features $e_1 = e^{**}$ and $n_1 = n^{**}$. When the debt level is above the threshold $d_1 > \bar{d}_1$, the

[26] In particular, we assume that the monetary policy is not used to target the efficient level of output, because this would exert upwards pressure on inflation (see Eq. (B.2) in Appendix B.1).

interest rate constraint binds and the unique equilibrium features a recession $e_1 < e^{**}$ and $n_1 < n^{**}$.

We next analyze the efficiency properties of the equilibrium. The analogue of the constrained planning problem (19) in this case is given by:

$$\max_{\left(\left(c_0^h\right)_h, d_1\right)} \sum_h \gamma^h \left(u\left(c_0^h\right) + \beta^h V^h\left(d_1^h, d_1\right)\right) \tag{B.9}$$

$$\text{such that} \qquad d_1 = d_1^b = -d_1^l \text{ and } \sum_h c_0^h = 2e^{**} = 2\left(n^{**} - v\left(n^{**}\right)\right).$$

In particular, the planner also respects the monopoly distortions at date 0 (as well as future dates). Solving problem (B.9), it follows that our main result, Proposition 2, is also unchanged after replacing e^* with e^{**}. The result generalizes because Eq. (15) continues to hold in this case, so that $\mathbf{d}e_1/\mathbf{d}d_1 = -1$ and leverage continues to exert negative aggregate demand externalities.

B.4 Separable preferences

This section generalizes our main result to a setting in which households have preferences that are linearly separable between consumption and labor. We also assume downward wage rigidities, as in Appendix B.2, which leads to a simpler analysis. However, the results also hold for price rigidities as in the baseline model with slightly different formulas (see Appendix A.5 in our NBER working paper version).

Suppose households have separable preferences, $u\left(c_t^h\right) - v\left(n_t^h\right)$. Let $\tau_t^{lab,h} = 1 - \frac{u'\left(c_t^h\right)}{v'\left(n_t^h\right)}$ denote the households' labor wedge, which captures the extent to which employment at date t is below (or above, if negative) its efficient level. Suppose the final good firms have flexible prices so that the nominal price equals the nominal wage level, $P_t = W_t$, as in Appendix B.2. However, nominal wages are rigid and cannot fall below a wage norm, $\overline{W}_t = W_{t-1}$, which we take to be last period's wage for simplicity. The labor supply of each household h satisfies:

$$W_t = \max\left\{\overline{W}_t, P_t \frac{v'\left(n_t^h\right)}{u'\left(c_t^h\right)}\right\}. \tag{B.10}$$

If the nominal price (and wage) level is sufficiently high, then $P_t \frac{v'\left(n_t^h\right)}{u'\left(c_t^h\right)} \geq \overline{W}_t$ holds so that the labor wedge is zero and the labor is supplied efficiently. Otherwise, the wage level satisfies $W_t = \overline{W}_t$ and then the aggregate labor supply is rationed and determined by aggregate demand, $\frac{n_t^b + n_t^l}{2} = \frac{c_t^b + c_t^l}{2}$. We also assume the symmetric rationing rule (that will be specified below) by which a unit decline in demand translates into a unit decline in the labor supply of each group.

First consider dates $t \geq 2$, at which the level of consumer debt is constant at the maximum permissible level $d_t = \phi$ and borrowers pay lenders a constant amount of interest $\left(1 - \frac{1}{1+r_{t+1}}\right)\phi = \left(1 - \beta^l\right)\phi$ at every date. In this case, the inflation targeting policy implements $\Pi_t = \Pi = 1$ along with a zero labor wedge, $\tau_t^{lab,h} = 0$. Households' labor supply is given by:

$$u'\left(n^{l*}+\left(1-\beta^{l}\right)\phi\right)=v'\left(n^{l*}\right),\text{ and }u'\left(n^{b*}-\left(1-\beta^{l}\right)\phi\right)=v'\left(n^{b*}\right).$$

The labor market is in equilibrium because $P_t = P_{t-1}$ also implies $W_t = W_{t-1} = \overline{W}_t$ (see Eq. (B.10)).

Now consider date 1. As in the main text, there is a threshold, $\bar{d}_1$, such that there is a liquidity trap only if $d_1 \geq \bar{d}_1$. First consider the case $d_1 = \bar{d}_1$, in which case $r_2 = 0$ but the outcomes are still efficient (in particular, the labor wage is zero). Consider lenders' Euler equation at zero interest rate, $u'\left(\bar{c}_1^l\right) = \beta^l u'\left(n^{l*} + \left(1-\beta^l\right)\phi\right)$, which determines their consumption $\bar{c}_1^l$. Their intratemporal condition, $u'\left(\bar{c}_1^l\right) = v'\left(\bar{n}^l\right)$, determines a corresponding employment level, $\bar{n}^l$. This in turn pins down the threshold debt level $\bar{d}_1$, from lenders' budget constraint, $\bar{c}_1^l = \bar{n}_1^l + \bar{d}_1 - \phi$. Borrowers' consumption is constrained, and their labor supply is the solution to their own intratemporal condition, $u'\left(\bar{n}_1^b - \left(\bar{d}_1 - \phi\right)\right) = v'\left(\bar{n}_1^b\right)$.

Next consider the equilibrium when $d_1 > \bar{d}_1$. In this case, the economy features a liquidity trap with $r_2 = 0$. The nominal wage (as well as the price) is equal to the norm, $W_1 = \overline{W}_1 = W_0$, and the labor supply is rationed. We assume labor supply is rationed symmetrically according to the rule:

$$n_1^l = \bar{n}_1^l - \Delta \text{ and } n_1^b = \bar{n}_1^b - \Delta, \tag{B.11}$$

where Δ denotes the size of the decline in demand and output (relative to the $d_1 = \bar{d}_1$ benchmark) to be determined. To solve for Δ, note that lenders' consumption is equal to $c_1^l = \bar{c}_1^l$. Using lenders' budget constraint in equilibrium, $c_1^l = \bar{n}_1^l - \Delta + d_1 - \phi$, we solve for the endogenous decline in output as:

$$\Delta = \bar{n}_1^l - \bar{c}_1^l + d_1 - \phi, \tag{B.12}$$

Eqs. (B.11) and (B.12) provide the analogue of Eq. (15) in the main text. Note that $\frac{\mathbf{d}n_1^l}{\mathbf{d}d_1} = -1$ and $\frac{\mathbf{d}n_1^b}{\mathbf{d}d_1} = -1$, which also implies $\frac{\mathbf{d}y_1}{\mathbf{d}d_1} = 0$. A unit increase in debt generates a unit decline in output, as well as each household's employment, similar to the main text.

The date 0 equilibrium is characterized by the Euler equations (16). Under conditions similar to those in Proposition 1, the equilibrium features $d_1 > \bar{d}_1$ and an anticipated recession.

We next analyze the efficiency properties of this equilibrium. First let $V^h\left(d_1^h, d_1\right)$ denote the utility of a household h conditional on entering date 1 with an individual level of debt d_1^h and an aggregate level of debt d_1. For the range $d_1 > \bar{d}_1$, we have:

$$\begin{aligned} V^b\left(d_{1,}^b d_1\right) &= u\left(n_1^b\left(d_1\right) - \left(d_1^b - \phi\right)\right) - v\left(n_1^b\left(d_1\right)\right) + \sum_{t=2}^{\infty}\left(\beta^b\right)^t\left(u\left(c^{b*}\right) - v\left(n^{b*}\right)\right), \\ \text{and } V^l\left(d_{1,}^l d_1\right) &= u\left(n_1^l\left(d_1\right) + \left(d_1^l - \phi\right)\right) - v\left(n_1^l\left(d_1\right)\right) + \sum_{t=2}^{\infty}\left(\beta^l\right)^t\left(u\left(c^{l*}\right) - v\left(n^{l*}\right)\right). \end{aligned}$$

The aggregate demand externalities are then given by:

$$\frac{\mathbf{d}V^h}{\mathbf{d}d_1} = \frac{\mathbf{d}n_1^h}{\mathbf{d}d_1}\left(u'\left(c_1^h\right) - v'\left(n_1^h\right)\right) = -u'\left(c_1^h\right)\tau_1^{lab,h}. \tag{B.13}$$

Hence, the externalities depend on the effect of debt on household's income (which is 1) as well as the labor wedge, $\tau_1^{lab,h} = 1 - \frac{v'\left(n_1^h\right)}{u'\left(c_1^h\right)} \in (0,1)$.

Next consider the ex-ante constrained planning problem, which is still given by (19). Combining the first order conditions with Eq. (B.13) implies

$$\frac{\beta^l u'\left(c_1^l\right)}{u'\left(c_0^l\right)} = \frac{\beta^b u'\left(c_1^b\right)}{u'\left(c_0^b\right)} - \left(\frac{\beta^b u'\left(c_1^b\right)}{u'\left(c_0^b\right)}\tau_1^{lab,b} + \frac{\beta^l u'\left(c_1^l\right)}{u'\left(c_0^l\right)}\tau_1^{lab,l}\right). \tag{B.14}$$

It follows that the analogues of Proposition 2 and its Corollaries 1 and 2 also hold for this case. Note, however, that the planner typically mitigates but does not fully avoid the recession. This is because $\tau_1^{lab,h} = 0$ when $d_1 = \overline{d}_1$, which implies that Eq. (B.14) will also have interior solutions with $d_1 > \overline{d}_1$ and $\tau_1^{lab,h} > 0$. We can also characterize the optimal tax rate on debt issuance, τ_0^b, which satisfies $\frac{1}{1-\tau_0^b} = 1 + \tau_1^{lab,b} + \tau_1^{lab,l}\frac{1}{1-\tau_0^b}$ A first order approximation around $\tau_0^b = 0$ then implies

$$\tau_0^b \simeq \tau_1^{lab,b} + \tau_1^{lab,l}. \tag{B.15}$$

As Eqs. (B.13) – (B.15) illustrate, when preferences are separable, the size of the inefficiency as well as the optimal intervention also depends on the labor wedge. Intuitively, raising aggregate demand improves social welfare to the extent to which labor is underutilized, as captured by the labor wedge.

The labor wedge does not appear in the formulas for the baseline model with GHH preferences, $u\left(\tilde{c}_t^h - v\left(n_t\right)\right)$, as these preferences generate an endogenous amplification mechanism. To see the amplification, recall that 1 unit of decline in debt in the baseline model increases the average *net income* by 1 unit, which means that it increases the average income by $1 + v\left(n_t\right)$ units [cf. Eq. (B.12)]. Intuitively, keeping the interest rate constant, any increase in labor costs $v\left(n_t\right)$ creates a further increase in demand and output (because unconstrained households' intertemporal substitution depends on net consumption, $c_t^h = \tilde{c}_t^h - v\left(n_t^h\right)$). This amplification ensures that the disutility of labor, as well as the labor wedge, does not appear in the optimal subsidy or tax formulas in the main text.